Lesson 4:

Extending MOEAs to Solve Complex Engineering Problems

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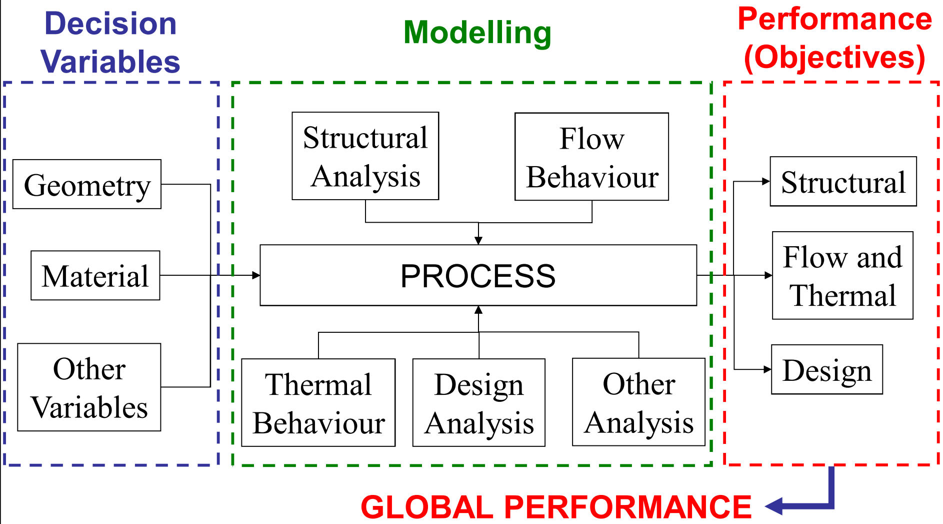
OUTLINE

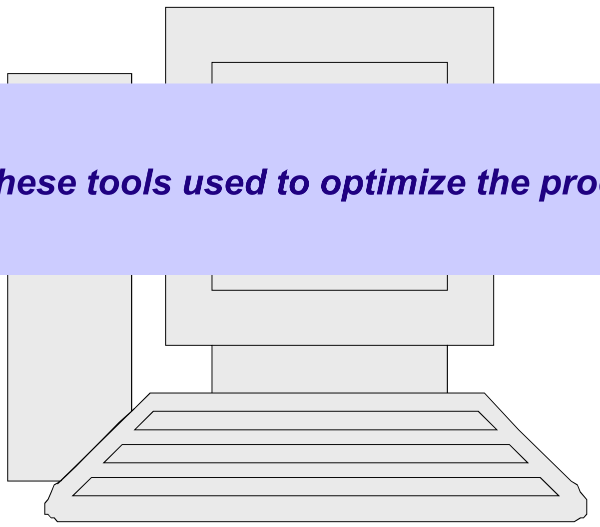
- Motivation
- Multi-Objective Evolutionary Algorithm
- Decision Making
- Robustness
- Hybrid Algorithms
- Number of Objectives Reduction
- Case Study
- Global Conclusions

MOTIVATION

Complicated mathematical models are able to describe adequately the (Engineering) processes to be optimized



MOTIVATION

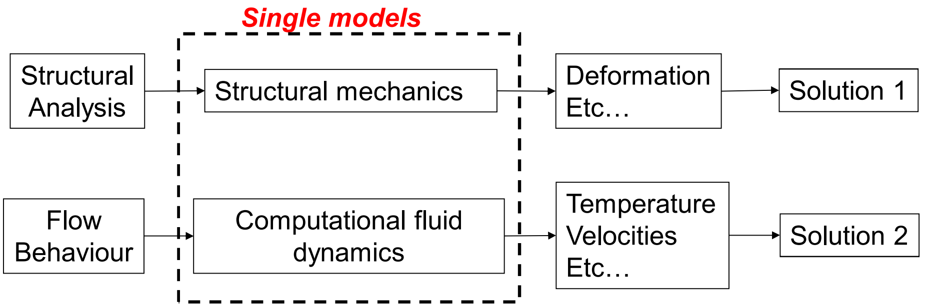


How are these tools used to optimize the processes?

MOTIVATION

Traditional approaches:

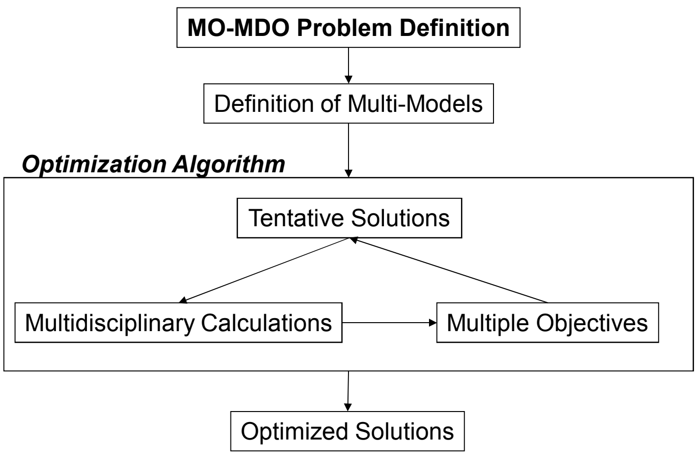
The traditional way of tackling this type of problems consists of using approximation and decomposition techniques to split a problem into simpler blocks and using simple specific models to give a more global representation of the problem.



MOTIVATION

Aim of our approach:

Global optimization procedure



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MOTIVATION

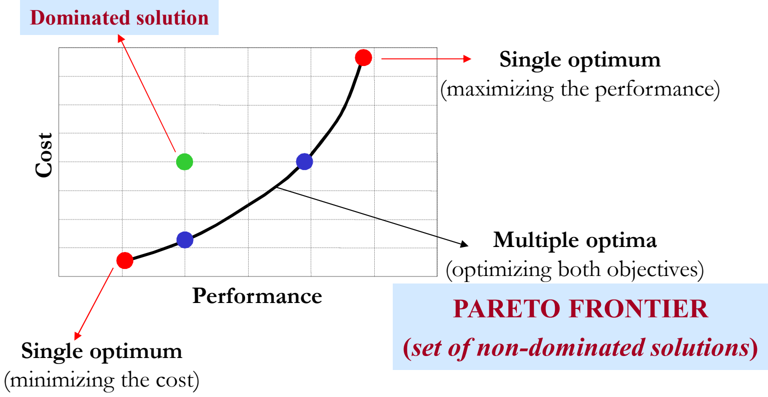
OBJECTIVE OF THE RESEARCH:

To implement a Multi-Objective Multidisciplinary Design and Optimization system (MO-MDO)

MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM

Most real optimization problems are multiobjective

Example: Simultaneous minimization of the cost and maximization of the performance of a specific system



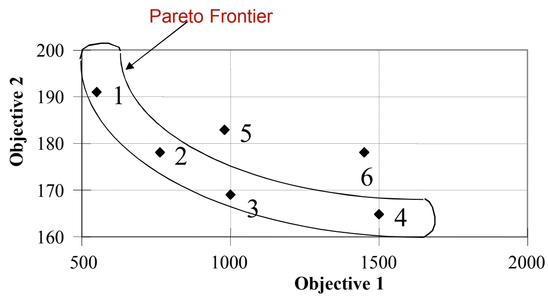
MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM

Method 1: **AGGREGATING FUNCTION**

$$FO_i = \sum_{j=1}^{q} w_j F_j$$

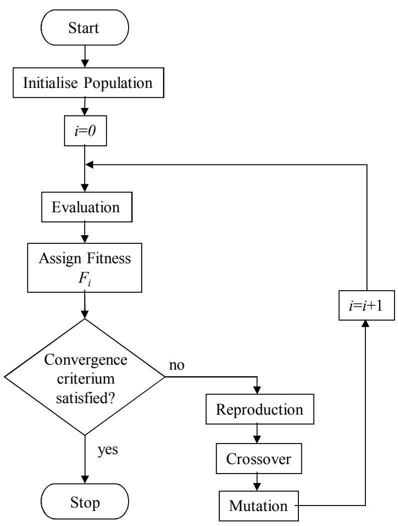
Method 2: **MULTIOBJECTIVE OPTIMIZATION USING EAs**

OBJECTIVE: *To obtain simultaneously several solutions along the Pareto frontier*



MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM

Evolutionary Algorithm



Population: ***set of individuals***

Initialization of population: ***random definition of all individuals of the population***

Evaluation: ***calculation of the values of the criteria using the modeling routine***

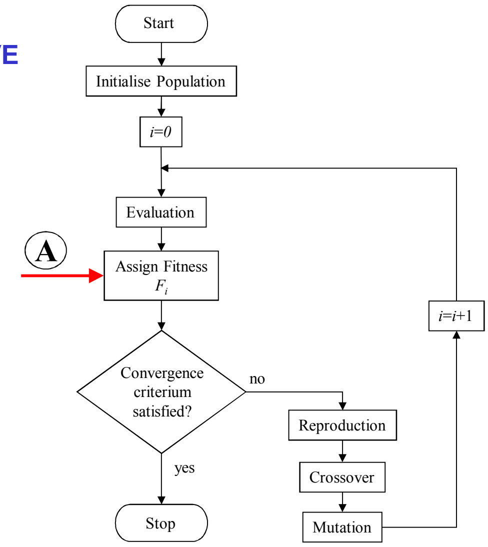
Fitness: ***calculation of a single value identifying the performance of individual***

Reproduction: ***selection of the best individuals for crossover and/or mutation***

Crossover/Mutation: ***methods to obtain new individuals for the next generation (i+1)***

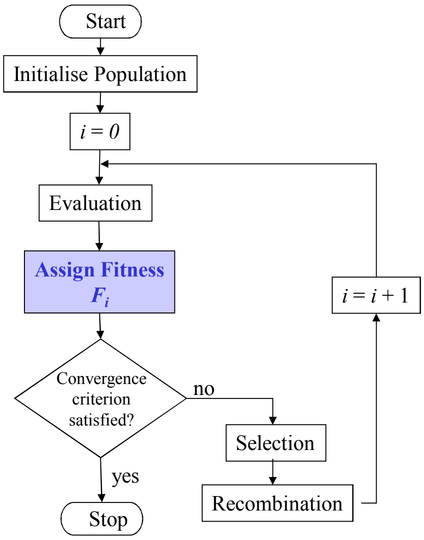
MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM

MULTI-OBJECTIVE OPTIMISATION



MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM

Reduced Pareto Set Genetic Algorithm (RPSGAe)



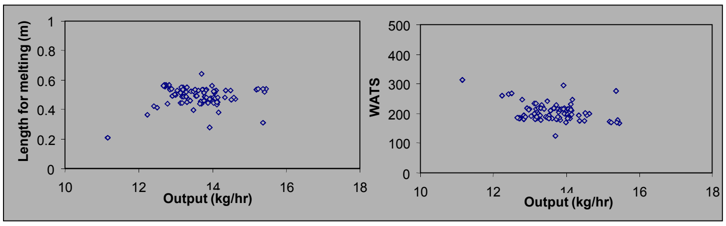
RPSGAe sorts the population individuals in a number of pre-defined ranks using a clustering technique, in order to reduce the number of solutions on the efficient frontier.

Details:

A. Gaspar-Cunha, J.A. Covas, - RPSGAe - A Multiobjective Genetic Algorithm with Elitism: Application to Polymer Extrusion, in Metaheuristics for Multiobjective Optimisation, Lecture Notes in Economics and Mathematical Systems, Gandibleux, X.; Sevaux, M.; Sörensen, K.; T'kindt, V. (Eds.), Springer, pp. 221-249, 2004.

MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM

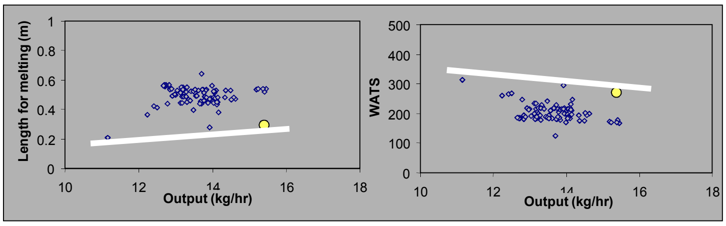
Multi-Objective Optimization - Example



Pareto curves in the criteria domain

MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM

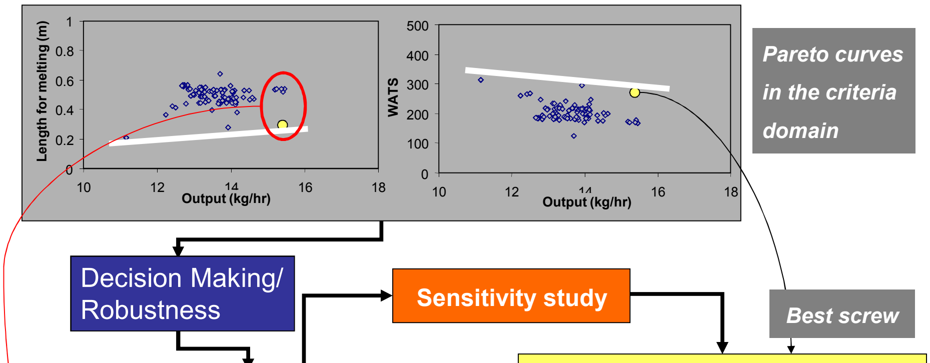
Multi-Objective Optimization - Example



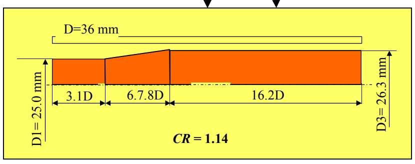
Pareto curves in the criteria domain

MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM

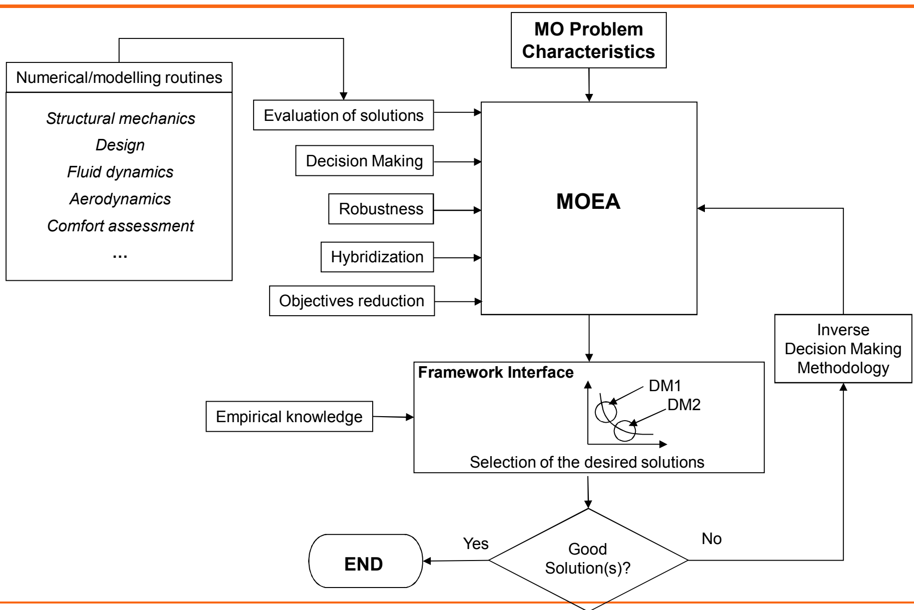
Multi-Objective Optimization - Example



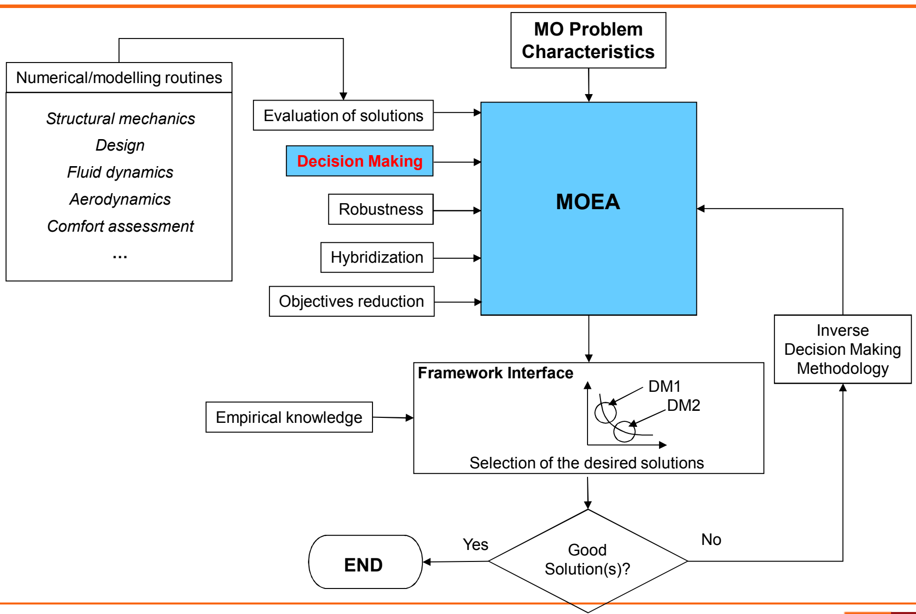
	L1	L2	L3	D1 (mm)	D3 (mm)	CR
Screw 1	3.1D	6.7D	16.2D	25.0	26.3	1.14
Screw 2	9.9D	5.5D	10.5D	25.6	26.1	1.05
Screw 3	10.4D	5.6D	10.0D	25.7	26.1	1.04
Screw 4	10.5D	5.6D	9.9D	25.7	26.1	1.05
Screw 5	10.5D	5.2D	10.3D	25.7	26.1	1.04



MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM

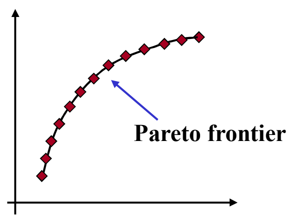
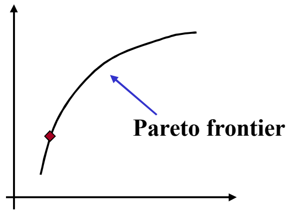


Decision in Multi-Objective Environment



DECISION MAKING

The solution of a Multi-Objective Optimization Problem results from:

	SEARCH PROCESS	DECISION PROCESS
Aim	*To obtain several solutions along the whole Pareto frontier*	*To obtain a single solution from the Pareto frontier*
How	*Using a Multi-objective Evolutionary Algorithm (MOEA)*	*Integrating the DM preference (criteria) information*
Result		

DECISION MAKING

Methods to deal with the DM preferences

A priori methods: The decision maker must specify her or his preferences, expectations and/or options before the optimization process takes place (aggregating function).

A posteriori methods: After the generation of the Pareto optimal set, the DM selects the most preferred among the alternatives taking into account his or her own preferences.

Interactive methods: Decision making and optimization occur at interleaved steps. At each step, partial preference information is supplied by the DM to the optimizer, which, in turn, generates better alternatives according to the information received.

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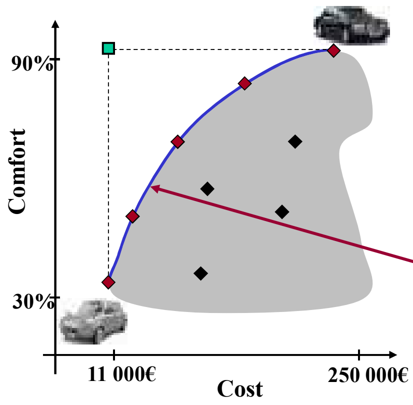
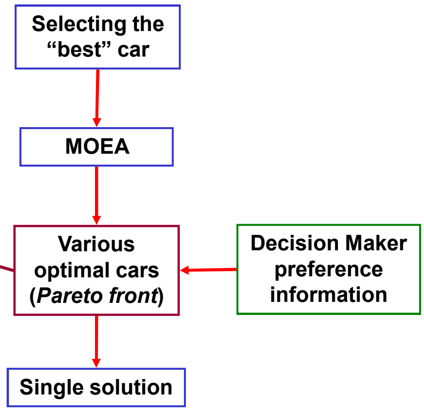
DECISION MAKING

METHODS TO DEAL WITH THE DM PREFERENCES

A Posteriori approach

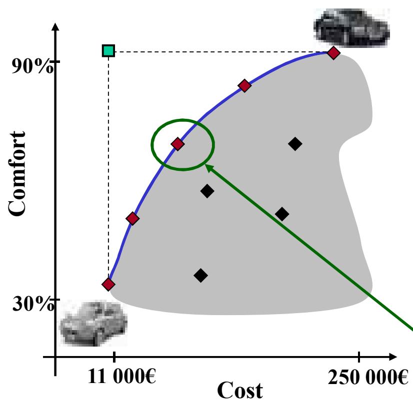


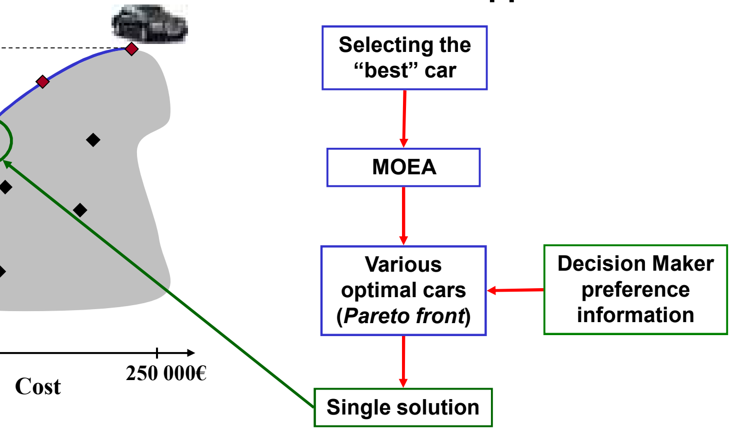


DECISION MAKING

METHODS TO DEAL WITH THE DM PREFERENCES

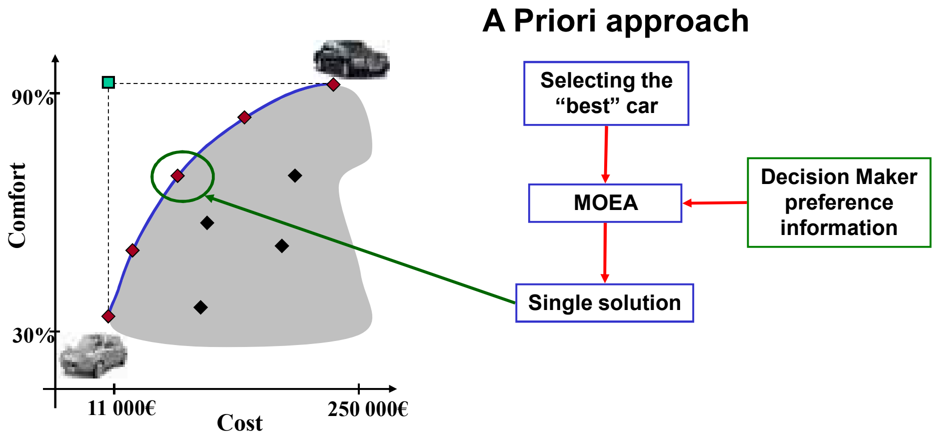
A Posteriori approach





DECISION MAKING

METHODS TO DEAL WITH THE DM PREFERENCES



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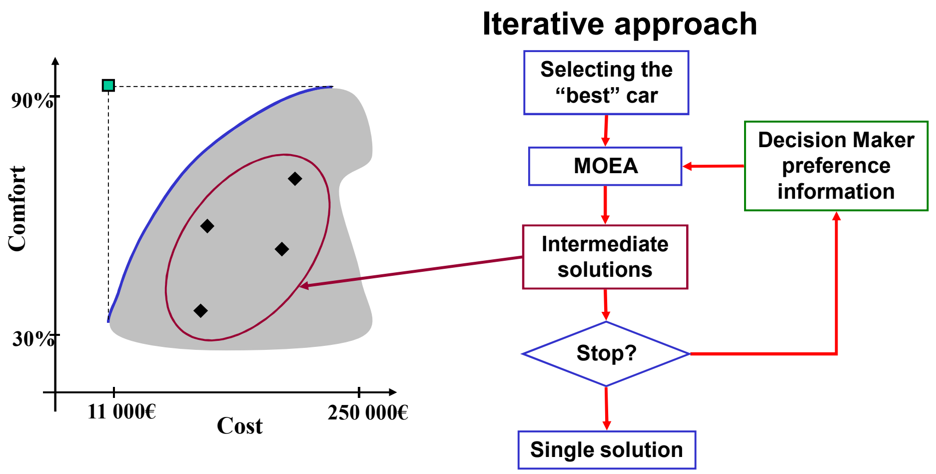
DECISION MAKING

METHODS TO DEAL WITH THE DM PREFERENCES



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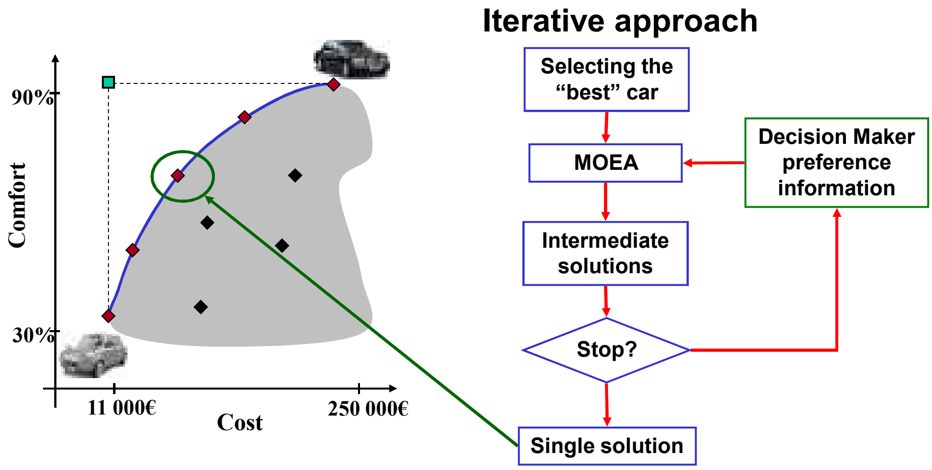
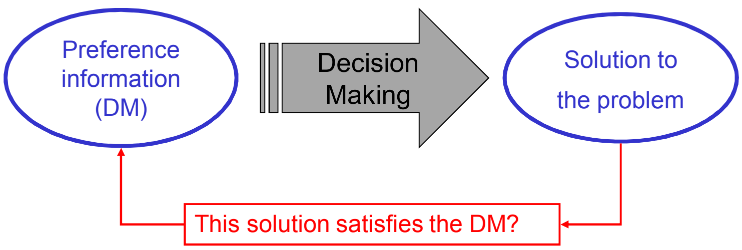
DECISION MAKING

METHODS TO DEAL WITH THE DM PREFERENCES



DECISION MAKING

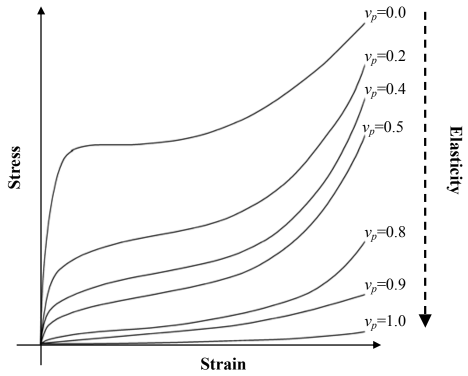
Correspondence between solutions and DM preferences



The applicability of decision methods depends strongly on the extent to which the parameter values defined by the DM (as an expression of his or her preferences) produce solutions corresponding to those preferences.

DECISION MAKING

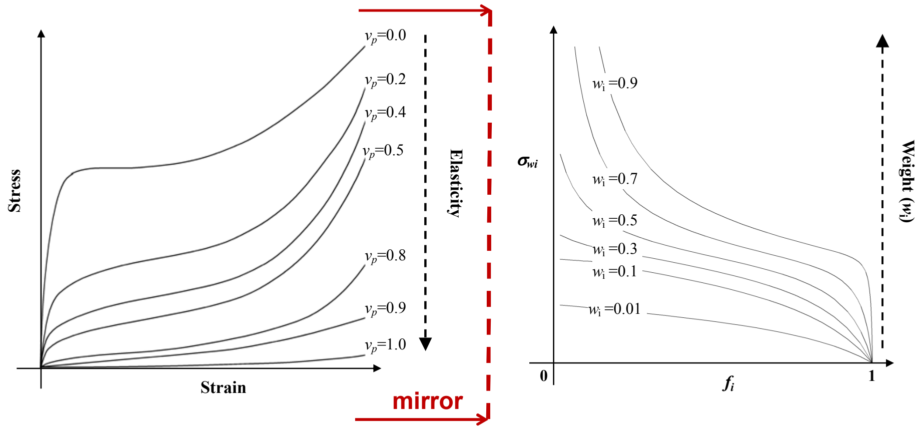
Rubber Elasticity Analogy



- This concept is based on the stress-strain behaviour of thermoplastic vulcanizate (TPV) materials.
- A TPV is formed by a cured elastomeric particles surrounded by a continuous thermoplastic matrix.
- When the volume fraction of the elastomeric particle is increased the elasticity increases .

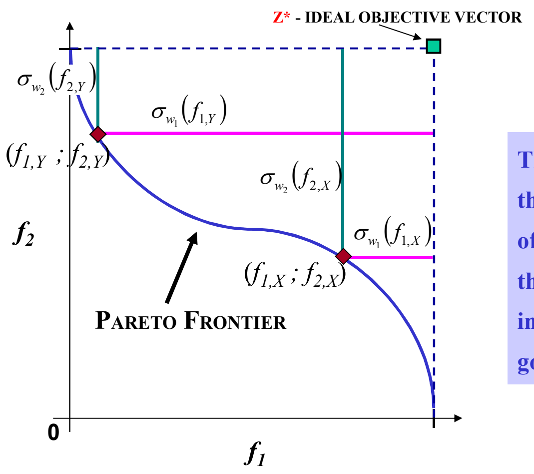
DECISION MAKING

Rubber Elasticity Analogy



DECISION MAKING

STRESS FUNCTION CONCEPT



σ_{wi}- Stress function (where w_i is the weight attributed to criterion i)

$$\sum_i w_i = 1, \qquad 0 \le w_i \le 1$$

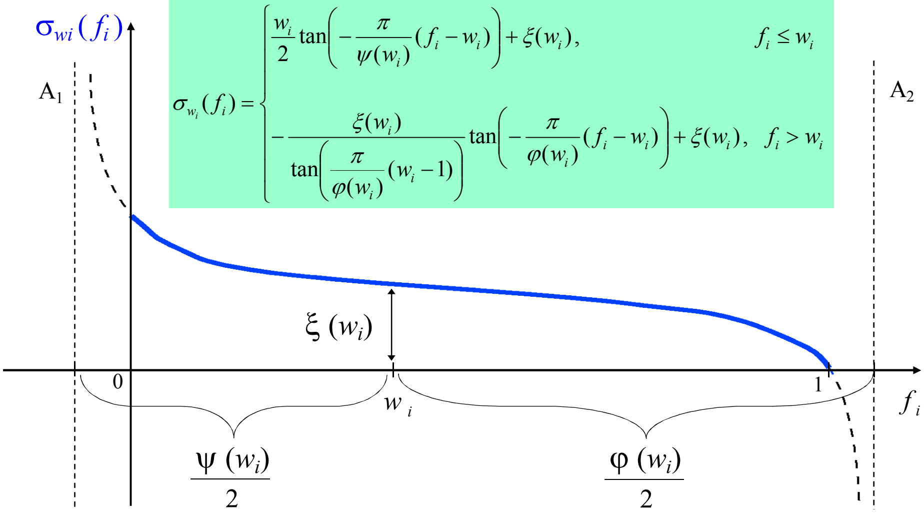
The stress function (σ_{w1}) increases as the distance between the coordinate of the ideal objective vector (1;1) and the coordinate of the solution increases, i.e., when this coordinate goes from $f_{1,X}$ to $f_{1,Y}$

$$\sigma_{w_1}(f_{1,X}) < \sigma_{w_1}(f_{1,Y})$$
$$\sigma_{w_2}(f_{2,X}) > \sigma_{w_2}(f_{2,Y})$$

DECISION MAKING

STRESS FUNCTION FORMULATION

$$\sigma_{w_i}(f_i) = \begin{cases} \dfrac{w_i}{2}\tan\left(-\dfrac{\pi}{\psi(w_i)}(f_i - w_i)\right) + \xi(w_i), & f_i \le w_i \\[2ex] -\dfrac{\xi(w_i)}{\tan\left(\dfrac{\pi}{\varphi(w_i)}(w_i - 1)\right)}\tan\left(-\dfrac{\pi}{\varphi(w_i)}(f_i - w_i)\right) + \xi(w_i), & f_i > w_i \end{cases}$$



DECISION MAKING

OBJECTIVE FUNCTION

$$\text{Minimize} \quad T(X) = \max_{i=1,\ldots,n} \left\{ \sigma_{w_i}\left(f_i(x)\right) \right\}$$

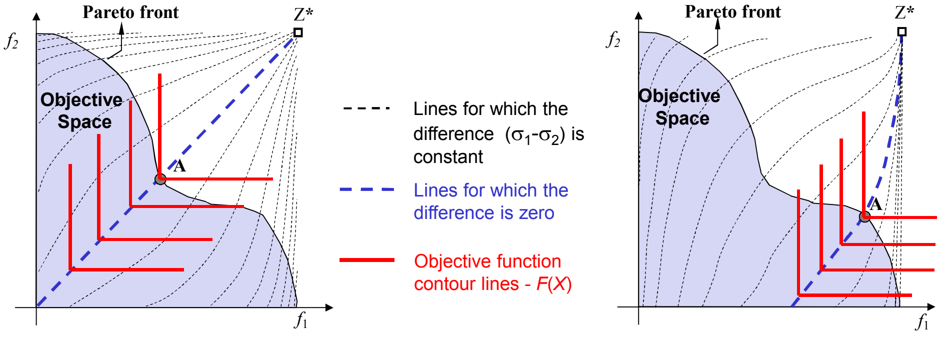
$$\text{Subject to} \quad x \in S$$

DECISION MAKING – *Weight Stress Function*

Objective Function

$$\text{Minimize} \quad F(x) = \max_{i=1,\ldots,n} \left\{ \sigma_{w_i}\left(f_i(x)\right) \right\}$$

$$\text{Subject to} \quad x \in S$$



Weight vector: (0.5, 0.5) **Weight vector: (0.8, 0.2)**

DECISION MAKING

- Weighted Sum
- Weight Tchebycheff Metric
- Reference Point Based EMO Approach
- Goal Programming

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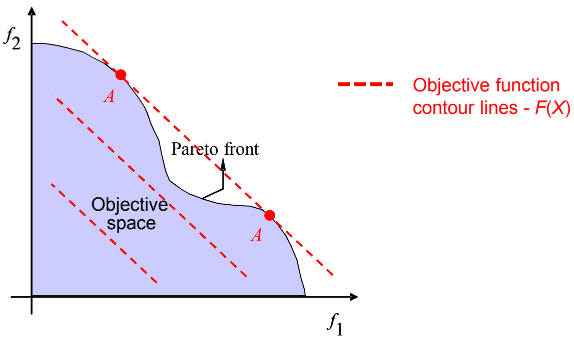
DECISION MAKING – *Weighted Sum*

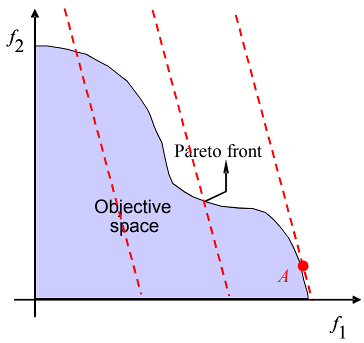
Objective Function

$$\text{Maximize} \quad F(x) = \sum_{i=1}^{n} w_i f_i(x)$$

$$\text{Subject to} \quad x \in S$$



Weight vector: (0.5, 0.5)



Weight vector: (0.8, 0.2)

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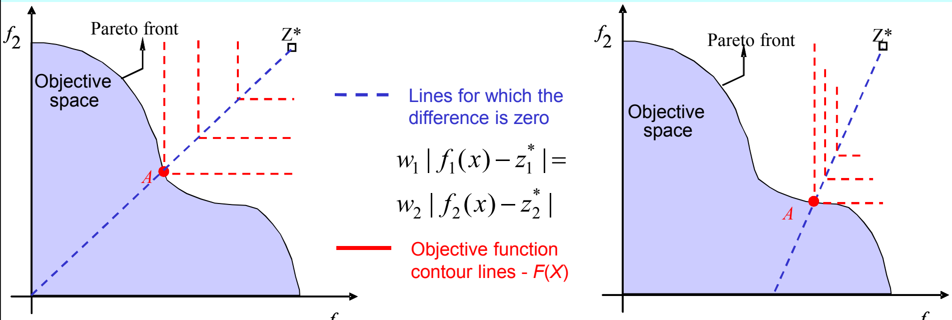
DECISION MAKING – *Weighted Tchebycheff Metric*

Objective Function

Minimize $F(x) = \max_{i=1,\ldots,n}\left\{ w_i \mid f_i(x) - z_i^* \mid \right\}$

Subject to $x \in S$



Weight vector: (0.5, 0.5) **Weight vector: (0.8, 0.2)**

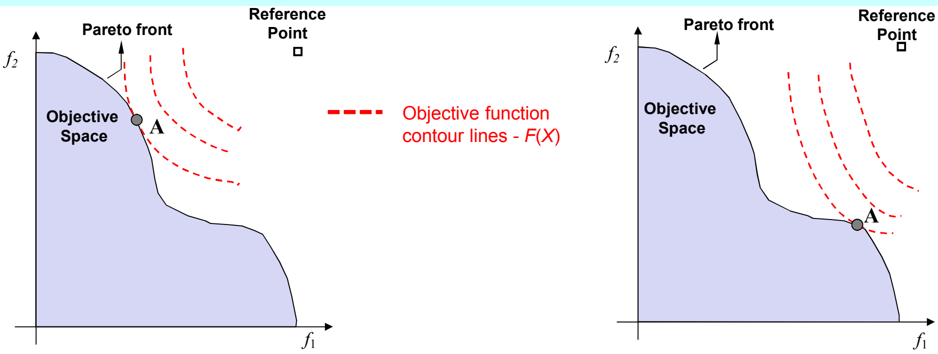
V.J. Bowman, On the Relationship of the Tchebycheff Norm and the Efficient Frontier of Multiple-Criteria Objectives, Multiple criteria Decision Making, Edited by H. Thiriez, S. Zionts, Lecture Notes in Economics and Mathematical Systems, Springer-Verlag, Berlin, pp. 76-85, 1976.

DECISION MAKING – *Reference Point Based EMO Approach*

Objective Function

Minimize $F(x) = d_{i,j}$

$$d_{ij} = \sqrt{\sum_{i=1}^{N} w_i \left(\frac{\bar{z}_i - f_i(X)}{f_i^{\max} - f_i^{\min}} \right)^2}$$



Weight vector: (0.5, 0.5) **Weight vector: (0.8, 0.2)**

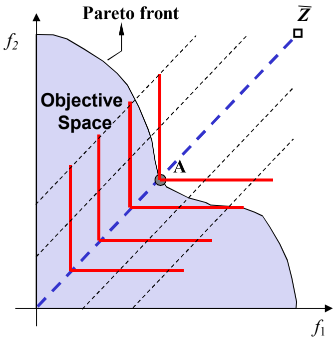
K. Deb, J. Sundar, U. Bhaskara and S. Chaudhuri, "Reference point based multi-objective optimization using evolutionary algorithms," International Journal of Computational Intelligence Research, vol.2, pp.273-286, 2006.

DECISION MAKING – Goal Programming

Objective Function

$$\text{Minimize} \quad F(x) = \max_{i=1,\ldots,N} \left[\overline{Z}_i - f_i(x) \right]$$

$$\text{Subject to} \quad x \in S$$

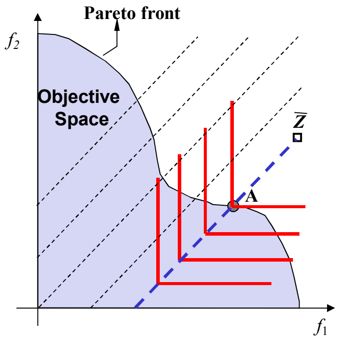


Goal: (1.0, 1.0)

- - - - Lines for which the difference is constant

- - - - Lines for which the difference is zero

—— Objective function contour lines - *F*(*X*)



Goal: (1.0, 0.6)

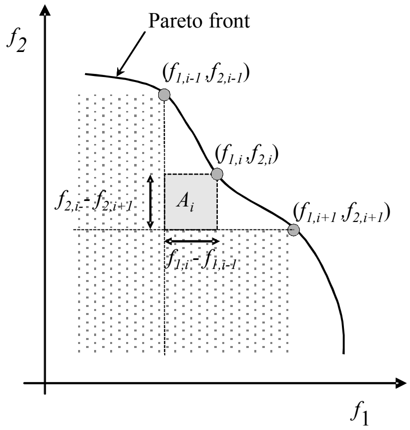
DECISION MAKING – Quality Assessment Methodology

OBJECTIVE: To develop a method able to assess the quality of the relation between the decision-model parameters (e.g., weight vector) and the resulting solutions

Ideally, the objective values of the set of non-dominated solutions selected from the Pareto frontier should reflect, in a consistent way, the changes made into the parameters vector

DECISION MAKING – *Quality Assessment Methodology*

Definition of a quality measure: "Hypervolume"



$$A_i = \left(f_{1,i} - f_{1,i-1}\right)\left(f_{2,i} - f_{2,i+1}\right)$$

1. Perform a uniform sampling of the weight vector ($w_{1,i}$);
2. For each pair ($w_{1,i}$; $w_{2,i}$) a different solution ($f_{1,i}$, $f_{2,i}$) is obtained;
3. Area A_i is computed for each solution;
4. The standard deviation of A_i is calculated;

Good performance implies low standard deviation!!!!

DECISION MAKING – *Test problems*

Assessing quality based on the hypervolume indicator

TP 1: $x \in [-2;6]$; Minimize; L=1; M=2

$$f_1(x) = x^2$$
$$f_2(x) = e^{|x|-5} + (6/5)\cos(2x) - 2{,}7x + 1$$

TP 2: $x \in [-1;1]$; Maximize; L=1; M=2

$$f_1(x) = -x^2 + 1$$
$$f_2(x) = \arctan(4x)$$

TP 3 (ZDT1): $x_i \in [0;1]$; Minimize; L=30; M=2

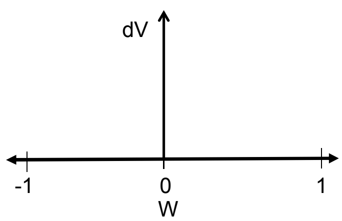
$$f_1(x_1) = x_1$$
$$f_2(x_2,\cdots,x_L) = g(x) \times \left(1 - \sqrt{f_1(x_1) / g(x)}\right)$$
$$\text{with, } g(x) = 1 + 9\frac{\sum_{l=2}^{L} x_l}{L-1}$$

TP 4 (ZDT2): $x_i \in [0;1]$; Minimize; L=30; M=2

$$f_1(x_1) = x_1$$
$$f_2(x_2,\cdots,x_L) = g(x) \times \left(1 - \left(f_1(x_1) / g(x)\right)^2\right)$$
$$\text{with, } g(x) = 1 + 9\frac{\sum_{l=2}^{L} x_l}{L-1}$$

In the case of the two objectives test problems (TP1 to TP4) a sequence of 300 distinct, equality spaced, values of the parameter w (∈ [-1, 1]) covering the interval [-1,1] was generated.

$$W = w_1 - w_2$$



DECISION MAKING – *Test problems*

Assessing quality based on the hypervolume indicator

TP 6 (DTLZ2): $x_i \in [0;1]$; Minimize; L=12; M=3

$$f_1(x) = (1+g(x)).\cos\left(x_1 \frac{\pi}{2}\right).\cos\left(x_2 \frac{\pi}{2}\right)$$

$$f_2(x) = (1+g(x)).\cos\left(x_1 \frac{\pi}{2}\right).\sin\left(x_2 \frac{\pi}{2}\right)$$

$$f_3(x) = (1+g(x)).\sin\left(x_1 \frac{\pi}{2}\right)$$

$$g(x) = \sum_{i=3}^{L} (x_i - 0.5)^2$$

TP 7: $x_i \in [0;1]$; Minimize; L=12; M=3

$$f_1(x) = 1-(1+g(x)).cos\left(x_1 \frac{\pi}{2}\right).cos\left(x_2 \frac{\pi}{2}\right)$$

$$f_2(x) = 1-(1+g(x)).cos\left(x_1 \frac{\pi}{2}\right).sin\left(x_2 \frac{\pi}{2}\right)$$

$$f_3(x) = 1-(1+g(x)).sin\left(x_1 \frac{\pi}{2}\right)$$

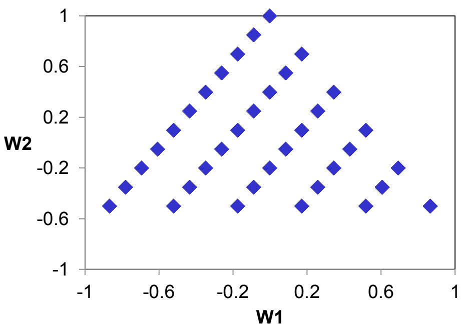
$$g(x) = \sum_{i=3}^{L} (x_i - 0.5)^2$$

3 objectives: a 2D triangular mesh with 4186 points was generated in a space (W1 and W2) correspondent to a linear combination of the three weights:

$$\mathrm{W1} = -\sin\left(\pi/3\right)w_1 + \sin\left(\pi/3\right)w_2$$

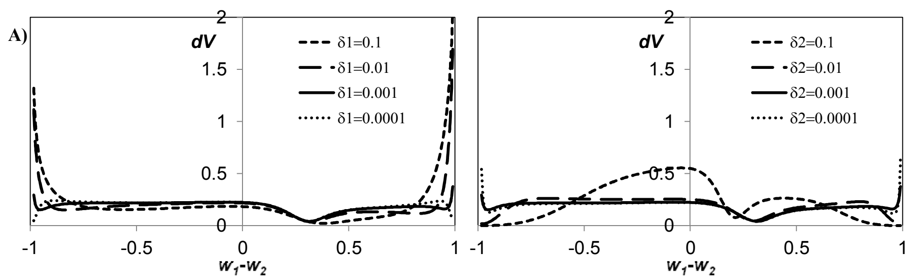
and :

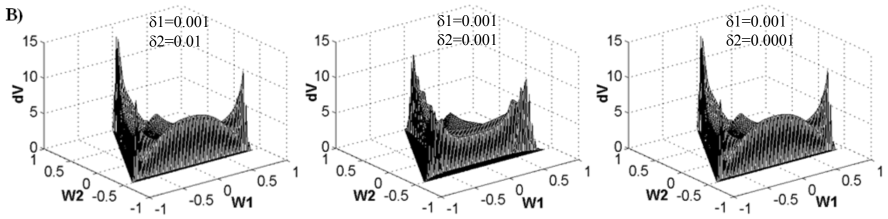
$$\mathrm{W2} = -\cos\left(\pi/3\right)w_1 - \cos\left(\pi/3\right)w_2 + w_3$$



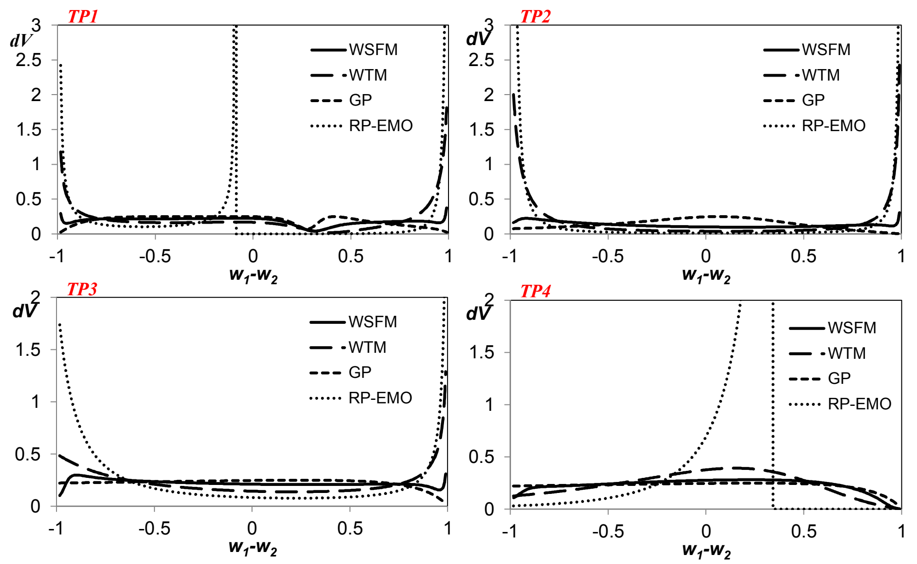
DECISION MAKING – *Test problems*

Influence of δ_1 and δ_2 for 2D and 3D test problems





DECISION MAKING – ***Results***



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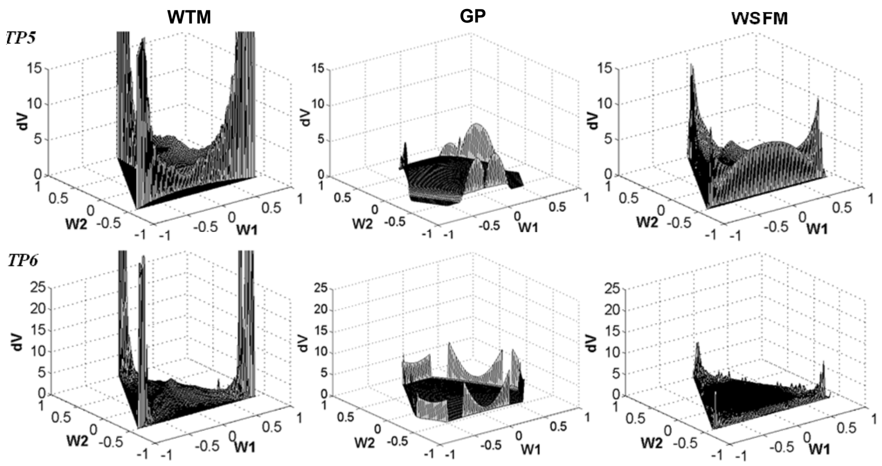
DECISION MAKING – ***Results***



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DECISION MAKING – Results

Comparison of the WSFM, WTM, GP and RP-EMO approaches based on the total Hypervolume and the ratio between standard deviation and the average of the uniformity measure for each method.

	RP-EMO		WTM		GP		WSFM	
	Hyp.	STD/avg	Hyp.	STD/avg	Hyp.	STD/avg	Hyp.	STD/avg
TP1	0.5970	1.074	0.6291	1.560	0.6340	**0.004**	**0.6346**	**0.218**
TP2	0.4656	17.843	0.6843	1.245	**0.8131**	**0.257**	0.8032	0.544
TP3	0.5358	3.113	0.5968	1.767	**0.6115**	**0.436**	0.6031	0.465
TP4	0.3291	**0.023**	0.3317	0.255	0.3317	0.436	**0.3318**	**0.242**
TP5	--	--	0.4108	2.785	0.3833	2.542	**0.4135**	**1.061**
TP6	--	--	0.4625	5.736	0.4150	2.478	**0.4673**	**0.601**

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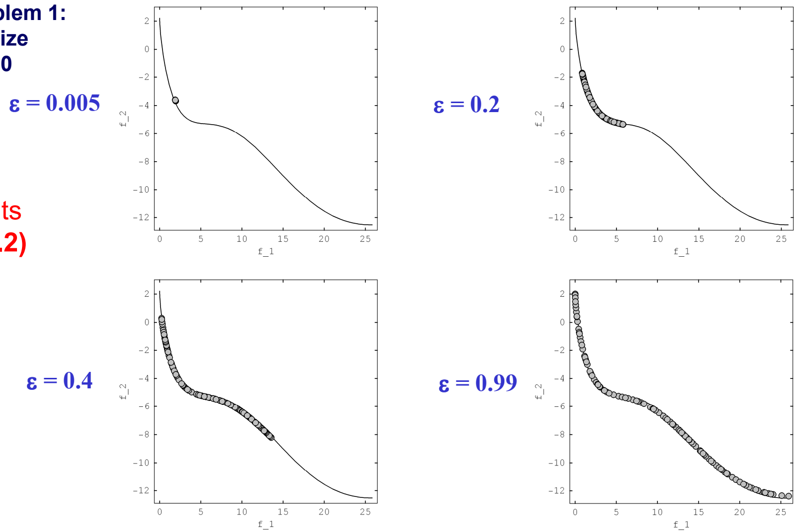
DECISION MAKING

INFLUENCE OF THE DISPERSION PARAMETER (ε)

Test problem 1:
Minimize
N = 100



Weights
(0.8, 0.2)

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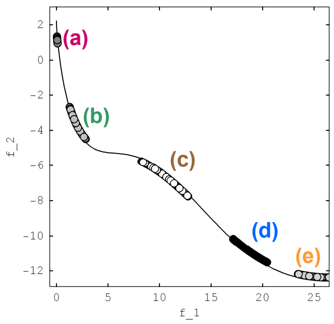
DECISION MAKING

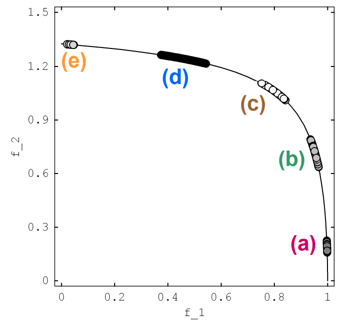
INFLUENCE OF THE WEIGHT VECTOR

Test problem 1:
Minimize
N = 100

$\varepsilon = 0.1$



Test problem 2:
Maximize
N = 100



Weights: (a) – (0.98, 0.02) (c) – (0.5, 0.5) (e) – (0.02, 0.98)

(b) – (0.8, 0.2) (d) – (0.2, 0.8)

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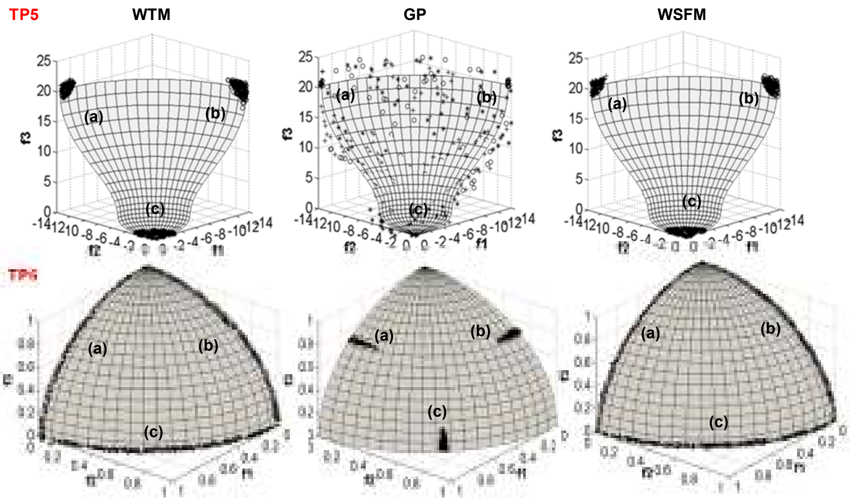
DECISION MAKING

Comparison between the WTM, GP and WSFM approaches for TP5 and TP6



Weight vectors: (a) (0,1,0), (b) (1,0,0) and (c) (0,0,1).

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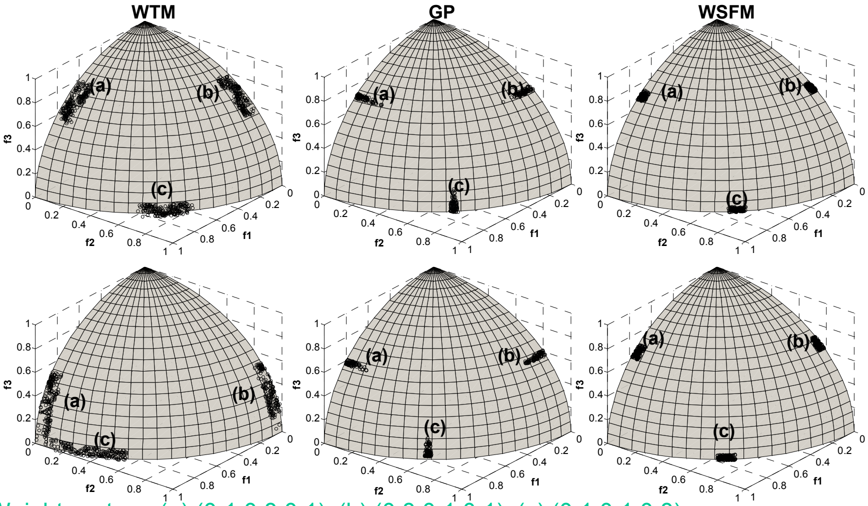
DECISION MAKING

Sensitivity comparison between the WTM, GP and WSFM approaches for TP6



Weight vectors: (a) (0.1,0.8,0.1), (b) (0.8,0.1,0.1), (c) (0.1,0.1,0.8),
(a*) (0.07,0.8,0.13), (b*) (0.8,0.07,0.13) and (c*) (0.07,0.13,0.8).

DECISION MAKING - Conclusions

- **A method able to take into account the preferences of the DM, based on the use of a stress function approach, was proposed.**
- **A measure allowing the evaluation of the uniformity of the results obtained for MOOP has been proposed and was used to compare the performance of three different decision making methods using seven different test problems.**
- **These results shown that the WSF method has the best performance on most of test problems tested (it has a better correspondence between the preferences of the DM and the solution obtained by the decision method).**

DECISION MAKING - References

1- J. C. Ferreira, C. M. Fonseca, A. Gaspar-Cunha, Selection of solutions in a multi-objective environment: polymer extrusion a case study, Evolutionary Methods For Design, Optimization And Control, P. Neittaanmäki, J. Périaux and T. Tuovinen (Eds.), CIMNE, Barcelona, Spain 2007.

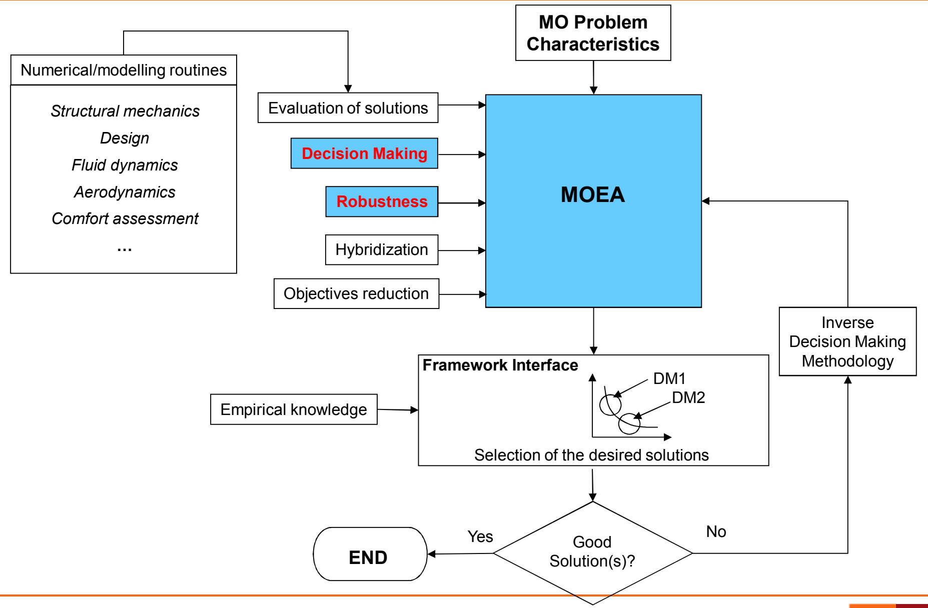
2- J. C. Ferreira, C. M. Fonseca, A. Gaspar-Cunha, Methodology to Select Solutions from the Pareto-Optimal Set: A Comparative Study, GECCO 2007, Genetic and Evolutionary Computation Conference, London, UK, July, 2007.

3- José C. Ferreira, Carlos M. Fonseca, A. Gaspar-Cunha, A New Methodology to Select the Preferred Solutions from the Pareto-optimal Set: Application to Polymer Extrusion, 10th Esaform Conference on Material Forming, Edited by E. Cueto and F. Chinesta, Zaragoza, Spain, April, 2007.

ROBUSTNESS

Robustness

ROBUSTNESS



ROBUSTNESS

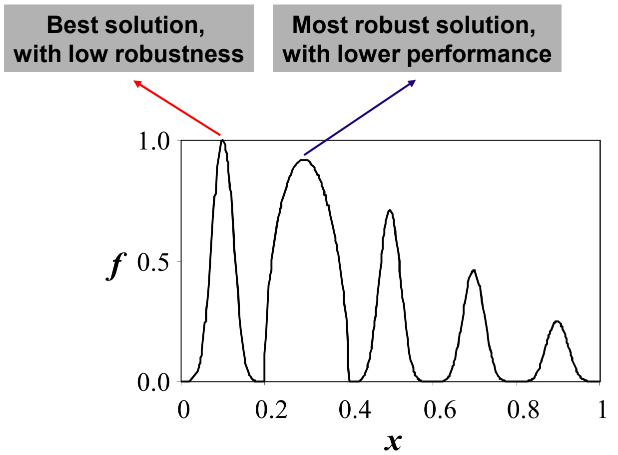
Objectives

i) To select an efficient method to quantify robustness

ii) To apply a robustness approach during multi-objective optimization problems using EMOAs

ROBUSTNESS

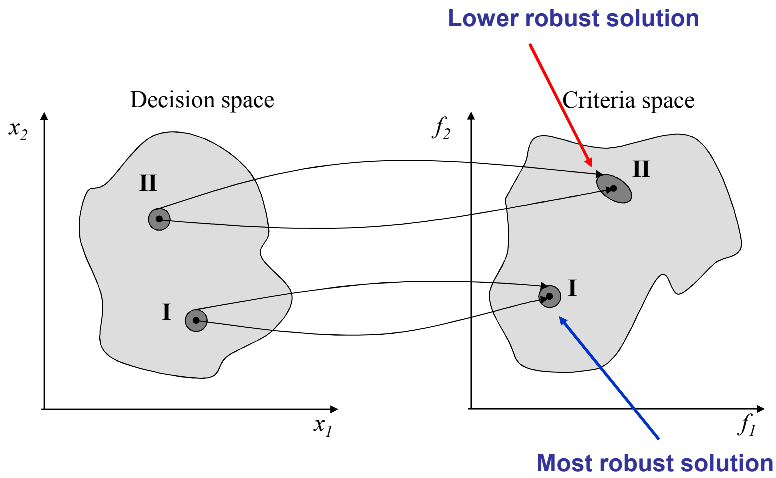
Concept of robustness for single objective functions



Sensitivity to variations of the design variables

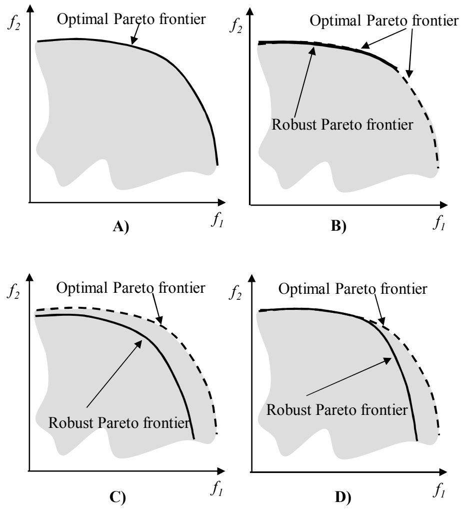
ROBUSTNESS

Concept of robustness for multi-objective functions



ROBUSTNESS

Optimal Pareto frontier versus robust Pareto frontier



ROBUSTNESS

Robustness can be taken into account by:

- Replacing *fm* by a measure of both its performance and expectation in the vicinity of the solution considered (**expectation measure)**.

- Considering an additional criterion for each of the *M* objective functions, which measures the variation of the original objective function around the vicinity of the design point considered (for example, variance) – **variance measure**.

Expectation Measures

Existing alternatives

1) $$F(x_i) = \frac{\sum_{j=1}^{n} w_j\, f\left(x_i + \Delta x_j\right)}{\sum_{j=1}^{n} w_j}$$

2) $$F(x_i) = \frac{f(x_i)}{\left|\dfrac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}\right|}$$

3) $$F(x_i) = \frac{\sum_{j=0}^{N'} f(x_j)}{N'}$$

4) $$F(x_i) = f(x_i)\left[\Phi\left(\frac{x+p}{g}\right) - \Phi\left(\frac{x-p}{g}\right)\right]$$

Proposed method

5) $$F(x_i) = \left(1 - \frac{\sum_{j=0}^{N} \left|\tilde{f}(x_j) - \tilde{f}(x_i)\right|}{N'}\right) f(x_i)$$

Maximization

$$\tilde{f}(x_i) = \frac{f(x_i) - f_{min}}{f_{max} - f_{min}}$$

Minimization

$$\tilde{f}(x_i) = 1 - \frac{f(x_i) - f_{min}}{f_{max} - f_{min}}$$

Variance Measures

Existing method

6) $$f^R = \frac{1}{N}\sum_{i=1}^{N} \frac{\sigma_f}{\sigma_{x_i}}$$

Proposed method

7) $$f_i^R = \frac{1}{N'}\sum_{j=0}^{N} \left|\frac{\tilde{f}(x_j) - \tilde{f}(x_i)}{x_j - x_i}\right|, \qquad d_{i,j} < d_{max}$$

ROBUSTNESS

Extending Robustness Measures to Multiple Objectives

1- The robustness of individual *i* can be calculated as the robustness average obtained for each criterion *m*:

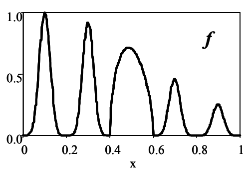
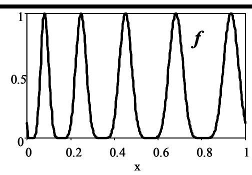
$$8)\quad f_i^{R1} = \frac{1}{M}\sum_{m=1}^{M} f_{i,m}^{R}$$

2- The robustness could be defined as the maximum of the robustness measures calculated for individual *i* in each criterion *m*:

$$9)\quad f_i^{R2} = \max_{m=1,\ldots,M} f_{i,m}^{R}$$

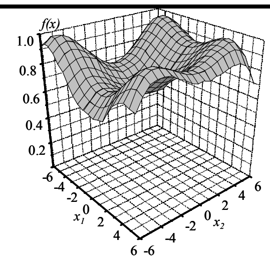
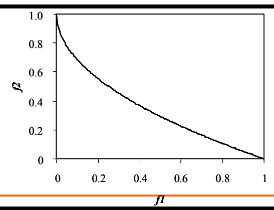
ROBUSTNESS

1st Study: Comparison of the different robustness measures

	Test Problems	x_i *aim* *L* *M*	Fitness function
1	$f(x) = \begin{cases} e^{-2\left(\frac{x-0.1}{0.8}\right)^2 \ln 2} \lvert\sin(5\pi x)\rvert^{0.5}, & a < x \le b \\ e^{-2\left(\frac{x-0.1}{0.8}\right)^2 \ln 2} \sin^6(5\pi x), & \text{otherwise} \end{cases}$ $f_1(x)$ with $a = 0.4$ and $b = 0.6$ $f_2(x)$ with $a = 0.2$ and $b = 0.4$ $f_3(x)$ with $a = 0.6$ and $b = 0.8$	[0;1] Max. L=1 M=1-3	
2	$f(x) = \sin^6\left(5\pi\left(x^e - 0.05\right)\right)$ $f_1(x)$ with $e = 0.75$ $f_2(x)$ with $e = 0.60$ $f_3(x)$ with $e = 0.45$	]0;1] Max. L=1 M=1-3	

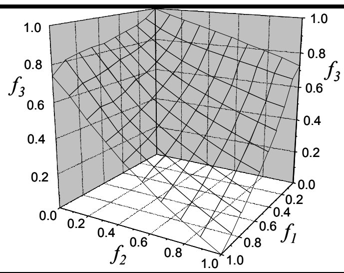
ROBUSTNESS

1st Study: Comparison of the different robustness measures

	Test Problems	x_i aim L M	Fitness function
3	$f(x)=\frac{2186-\left(x_1^2+x_2-c\right)^2-\left(x_1+x_2^2-d\right)^2}{2186}$	[-6;6] Max. L=2 M=1	
4	$f_1(x_1)=x_1$ $f_2(x_2,\cdots,x_L)=g\times\left(1-\sqrt{f_1/g}\right)$ with, $g(x_2,\cdots,x_L)=1+9\frac{\sum_{l=2}^{L}x_l}{L-1}$	[0;1] Min L=30 M=2	

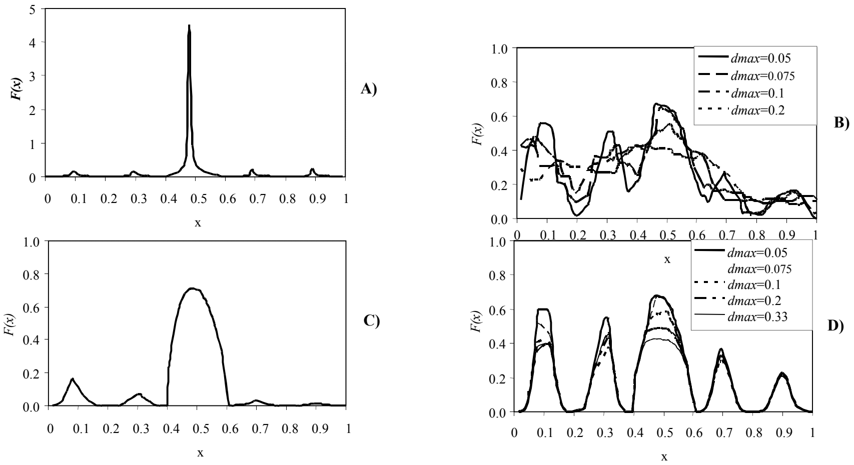
ROBUSTNESS

1st Study: Comparison of the different robustness measures

	Test Problems	x_i aim L M	Fitness function
5	$f_1(x_1)=x_1$ $f_2(x_2)=x_2$ $f_3(x_3,\cdots,x_L)=g\times\left(1-\sqrt{\frac{f_1\,f_2}{g}}\right)$ with, $g(x_3,\cdots,x_L)=1+9\frac{\sum_{l=3}^{L}x_l}{L-1}$	[0;1] Min L=30 M=3	

ROBUSTNESS

Individual robustness expectation measures



Expectation measures for test problem 1, using equations: A) 2; B) 3; C) 4 and D) 5.

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ROBUSTNESS

Comparative performance of the expectation measures

Equation	Clear definition	Independence of parameters	Efficiency
2	Yes	Yes	Small
3	No	No	Medium
4	Yes	No	Small
5	Yes	Yes	High

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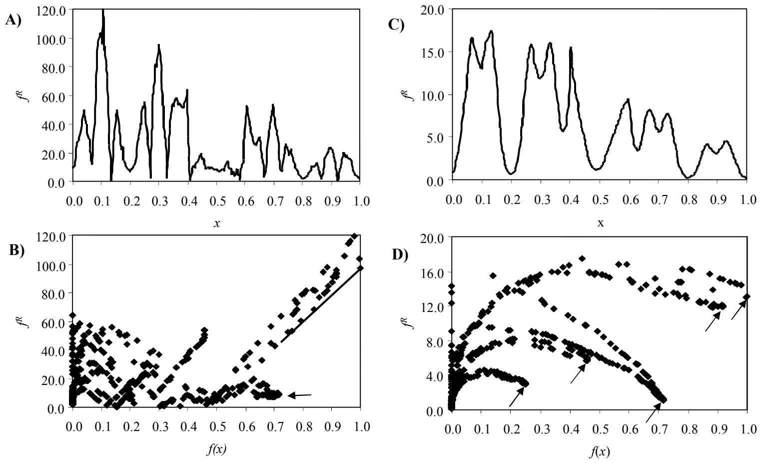
ROBUSTNESS

Individual robustness variance measures



Robustness of test problem 1, using equations 6: (A) robustness *vs. x*, B) Pareto-curves; and 7: (C) robustness *vs. x*, D) Pareto-curves

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ROBUSTNESS

Comparative performance of the variance measures

Equation	Clear definition	Independence of parameters	Efficiency	Clear definition of maxima
6	No	Yes	Medium	No
7	Yes	Yes	High	Yes

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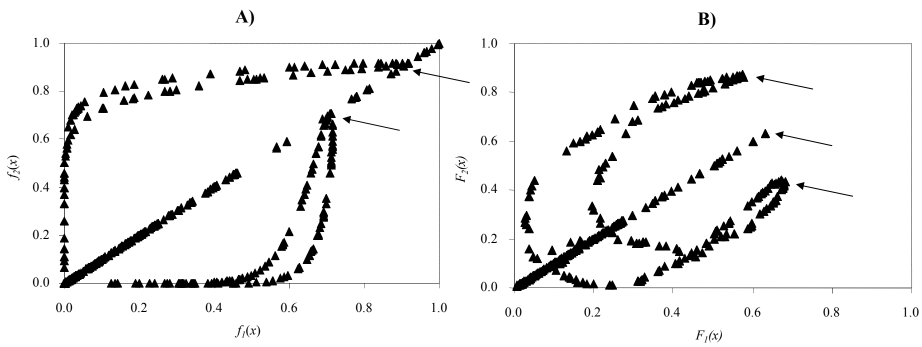
ROBUSTNESS

Combined robustness measures



Test problem 1 with two criteria, using: A) original fitness functions; B) trade-off between expectation measures

ROBUSTNESS

Combination 1

Original functions plus individual robustness measures $f_i(x) + f_i^R(x)$

Combination 2

Original functions plus average robustness measure $f_i(x) + f_i^{R1}(x)$

Combination 3

Original functions plus average robustness measure $f_i(x) + f_i^{R2}(x)$

Combination 4

Original functions $f_i(x)$

Combination 5

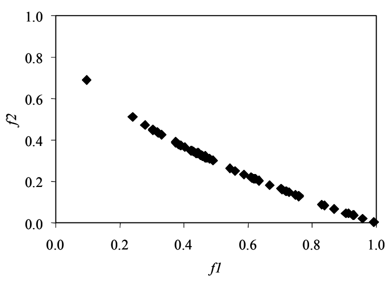
Expectation functions $F_i(x)$

ROBUSTNESS

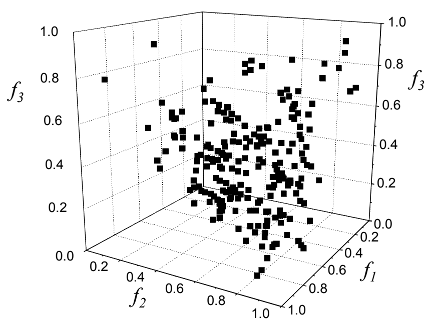
Combination	Problem	N. of criteria	No. of peaks (%)		Efficiency (%)		Precision (%)		Global (%)
1	1	2	57	76	67	92	79	81	83
		3	100		100		99		
	2	2	100		100		98		
		3	47		100		46		
2	1	2	43	59	100	88	59	64	70
		3	75		100		80		
	2	2	50		50		50		
		3	67		100		66		
3	1	2	29	54	67	85	38	58	66
		3	75		75		79		
	2	2	60		100		60		
		3	53		100		53		
4	1	2	14	21	33	52	20	26	33
		3	13		25		20		
	2	2	10		50		19		
		3	47		100		47		
5	1	2	14	18	33	40	19	21	26
		3	13		25		20		
	2	2	10		0		12		
		3	33		100		33		

ROBUSTNESS

Optimization of Multi-objective problems



Test problem 4



Test problem 5

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ROBUSTNESS – Conclusions

- **A measure of the robustness of solutions was introduced in a multi-objective evolutionary algorithm;**
- **Two types of robustness measures are defined, expectation and variance;**
- **The approach enabled the use of various combinations of criteria that were assessed using benchmark problems;**
- **The most performing uses the original criteria combined with an equal number of new criteria quantifying robustness through the variance measure proposed in this work;**
- **The methodology was successfully applied to a number of multi-objective problems.**

ROBUSTNESS

Robustness can be taken into account by:

- Replacing *fm* by a measure of both its performance and expectation in the vicinity of the solution considered (**expectation measure**).

$$F(x_i) = \left(1 - \frac{\sum_{j=0}^{N} \left| \tilde{f}(x_j) - \tilde{f}(x_i) \right|}{N'} \right) f(x_i)$$

- Considering an additional criterion for each of the M objective functions, which measures the variation of the original objective function around the vicinity of the design point considered (for example, variance) – **variance measure**.

$$f_i^R = \frac{1}{N'} \sum_{j=0}^{N} \left| \frac{\tilde{f}(x_j) - \tilde{f}(x_i)}{x_j - x_i} \right|, \qquad d_{i,j} < d_{max}$$

ROBUSTNESS

The robustness of the solution i can be calculated by the following equation:

$$R(i)=\frac{1}{N'}\sum_{j=0}^{N}\frac{\left\|f(x_j)-f(x_i)\right\|_2}{\left\|x_j-x_i\right\|_2}, \qquad d_{i,j}<d_{\max}$$

where N represents the number of neighbours (x_j) of x_i such that the distance ($d_{i,j}$) between both is not greater than d_{max}

$$m(i)=\sum_{j=1}^{N} sh(d_{ij})$$

$$\tilde{F}(i)=Rank(i)+(1-\varepsilon')\frac{R(i)}{R(i)+1}+\varepsilon'\frac{m(i)}{m(i)+1}$$

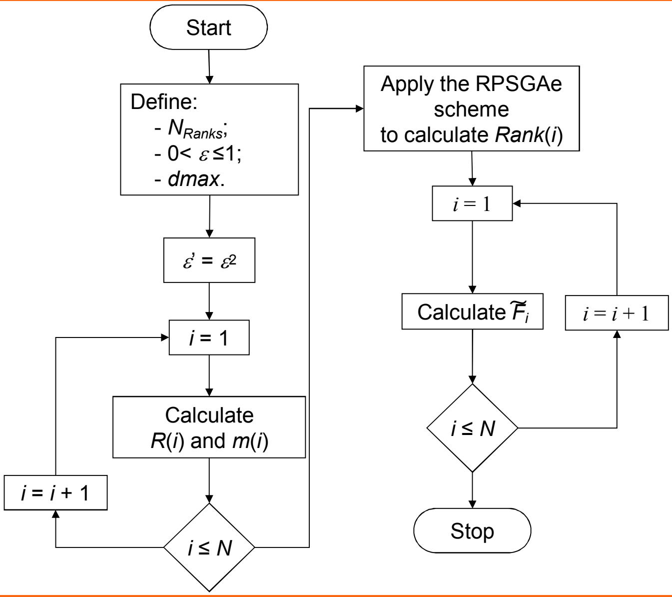
ROBUSTNESS

1- Random initial population (internal)
2- Empty external population
3- while not Stop-Condition do
  - a- Evaluate internal population
  - b- Calculate expectation and/or robustness measures
  - c- Calculate niche count
  - d- Calculate the Ranking of the individuals using the RPSGAe
  - e- Calculate the global Fitness
  - f- Copy the best individuals to the external population
  - g- if the external population becomes full
    - Apply the RPSGAe to this population
    - Copy the best individuals to the internal population
  - end if
  - h- Select the individuals for reproduction
  - i- Crossover
  - j- Mutation

end while

A. Gaspar-Cunha, J.A. Covas, Robustness in Multi-Objective Optimization using Evolutionary Algorithms, Computational Optimization and Applications, 39, pp. 75-96, 2008.

ROBUSTNESS



ROBUSTNESS

The following calculation steps must be carried out:

1. The robustness routine starts with the definition of the number of ranks (N_{ranks}), the span of the Pareto frontier to be obtained ($\varepsilon \in [0,1]$) and the maximum radial distance to each solution to be considered in the robustness calculation (d_{max});
2. To reduce the sensitivity of the algorithm to small values of the objective functions, the dispersion parameter is changed as $\varepsilon' = \varepsilon^2$;
3. For each individual, *i*, robustness , *R*(*i*), and niche count, *m*(*i*) , are determined;
4. The RPSGA algorithm is applied, with some modifications introduced to calculate *Rank*(*i*);
5. For each solution, *i*, the new fitness is calculated.

ROBUSTNESS - Test Problems

TP 1: x ∈[-2;6]; Minimize; *L*=1; *M*=2.

$$f_1(x) = x^2$$
$$f_2(x) = e^{|x|-5} + (6/5)\cos(2x) - 2{,}7x + 1$$

TP 2: x ∈ [0;5]; Maximize; *L*=1; *M*=2.

$$f_1(x) = x$$
$$f_2(x) = -5x + \cos(4x)$$

TP 3 (ZDT1): x_i ∈[0;1]; Minimize; *L*=30; *M*=2; Deb, Pratapat et al., 2002.

$$f_1(x_1) = x_1$$
$$f_2(x_2, \cdots, x_L) = g(x) \times \left(1 - \sqrt{f_1(x_1) / g(x)}\right)$$
$$\text{with, } g(x) = 1 + 9\frac{\sum_{l=2}^{L} x_l}{L-1}$$

TP 4 (ZDT2): x_i ∈[0;1]; Minimize; *L*=30; *M*=2; Deb, Pratapat et al., 2002.

$$f_1(x_1) = x_1$$
$$f_2(x_2, \cdots, x_L) = g(x) \times \left(1 - \left(f_1(x_1) / g(x)\right)^2\right)$$
$$\text{with, } g(x) = 1 + 9\frac{\sum_{l=2}^{L} x_l}{L-1}$$

ROBUSTNESS - Test Problems

TP 5 (ZDT3): x_i ∈[0;1]; Minimize; *L*=30; *M*=2; Deb, Pratapat et al., 2002.

$$f_1(x_1) = x_1$$
$$f_2(x_2, \cdots, x_L) = g(x) \times \left(1 - \sqrt{x_1 / g(x)} - \frac{x_1}{g(x)}\sin(10\pi x_1)\right)$$
$$\text{with, } g(x) = 1 + 9\frac{\sum_{l=2}^{L} x_l}{L-1}$$

TP 6: x_1 ∈[0;2π]; x_2 ∈[0;5]; Minimize; *L*=2; *M*=3.

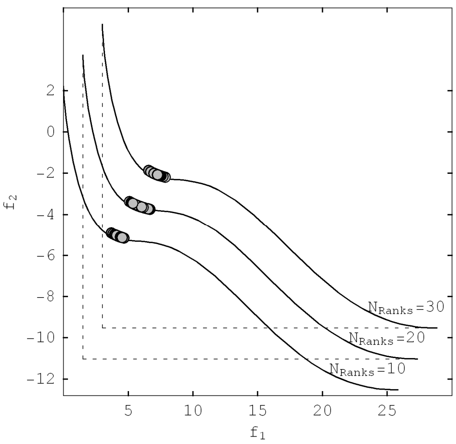
$$f_1(x) = sin(x_1).g(x_2)$$
$$f_2(x) = cos(x_1).g(x_2)$$
$$f_3(x) = x_2^2$$
$$\text{with, } g(x_2) = e^{x_2(x_2-5)} - \frac{6}{5}sin(2x_2) - 2{,}7x_2 - 1$$

TP 7 (DTLZ2): x_i ∈[0;1]; Minimize; *L*=12; *M*=3; Deb, Thiele et al., 2002.

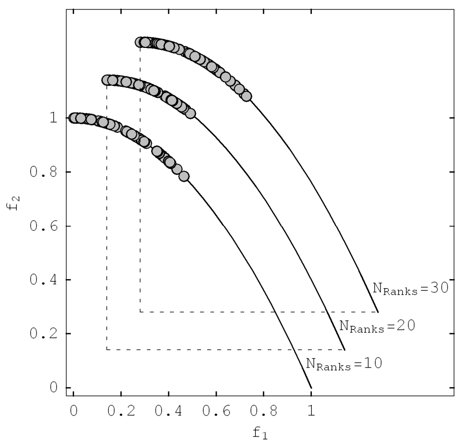
$$f_1(x) = (1 + g(x)).cos\left(x_1 \frac{\pi}{2}\right).cos\left(x_2 \frac{\pi}{2}\right)$$
$$f_2(x) = (1 + g(x)).cos\left(x_1 \frac{\pi}{2}\right).sin\left(x_2 \frac{\pi}{2}\right)$$
$$f_3(x) = (1 + g(x)).sin\left(x_1 \frac{\pi}{2}\right)$$
$$\text{with, } g(x) = \sum_{i=3}^{L}(x_i - 0.5)^2$$

ROBUSTNESS

Influence of N_{ranks}



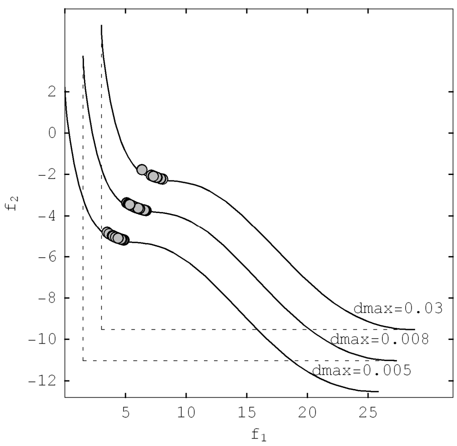
TP1



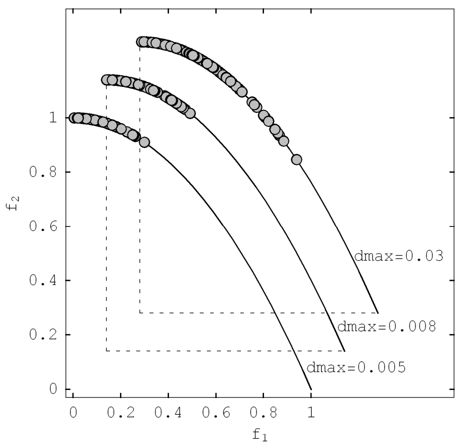
TP4

ROBUSTNESS

Influence of d_{max}



TP1

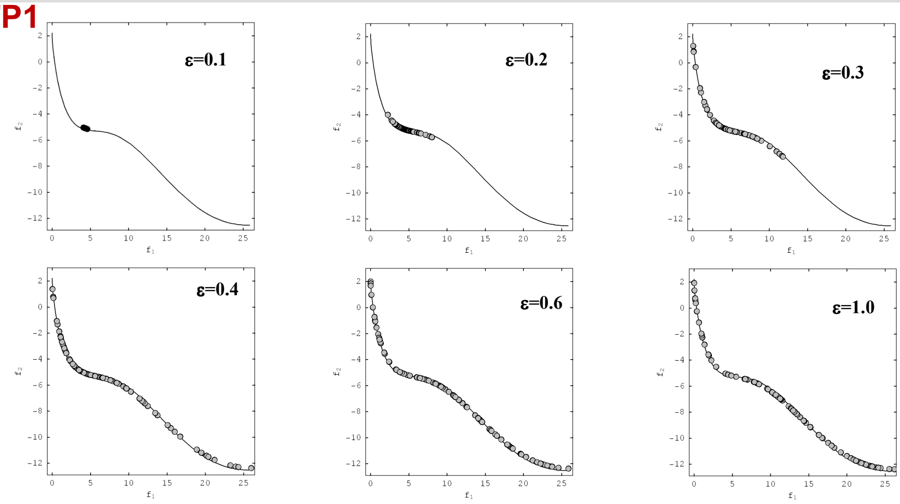


TP4

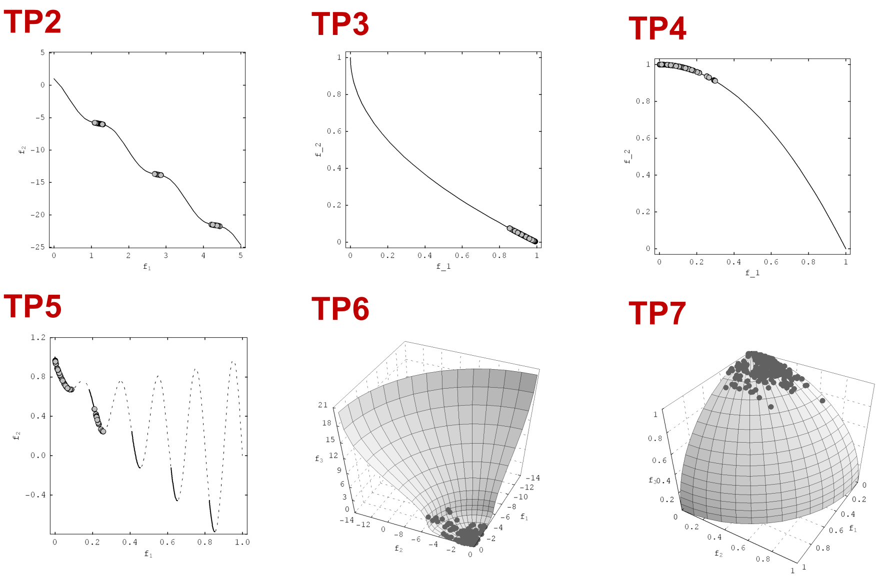
ROBUSTNESS

New methodology based on the concept of the dispersion parameter developed for Decision making

TP1



ROBUSTNESS



ROBUSTNESS - References

1- J. Ferreira, C. Fonseca, J.A. Covas and A. Gaspar-Cunha, Evolutionary Multi-Objective Robust Optimization, in Advances in Evolutionary Algorithms, ISBN 978-3-902613-32-5, i-TECH Education and Publishing (http://www.books.i-techonline.com/evo.html), 2008.

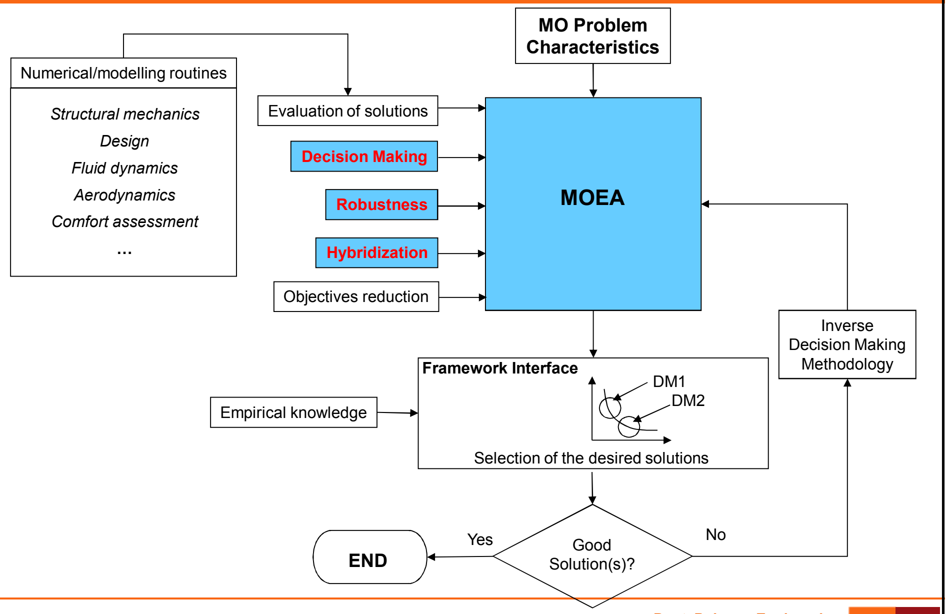
2- A. Gaspar-Cunha, J.A. Covas, Robustness in Multi-Objective Optimization using Evolutionary Algorithms, Computational Optimization and Applications, 39, pp. 75-96, 2008.

3- A. Gaspar-Cunha, J.A. Covas, Robustness using Multi-Objective Evolutionary Algorithms, WSC10 – 10th Online World Conference in Soft Computing in Industrial Applications, (http://www.cranfield.ac.uk/wsc10/), ISBN: 3-540-29123-7, pp. 189-193, 2005.

HYBRID ALGORITHMS

Hybrid algorithms

HYBRID ALGORITHMS



HYBRID ALGORITHMS

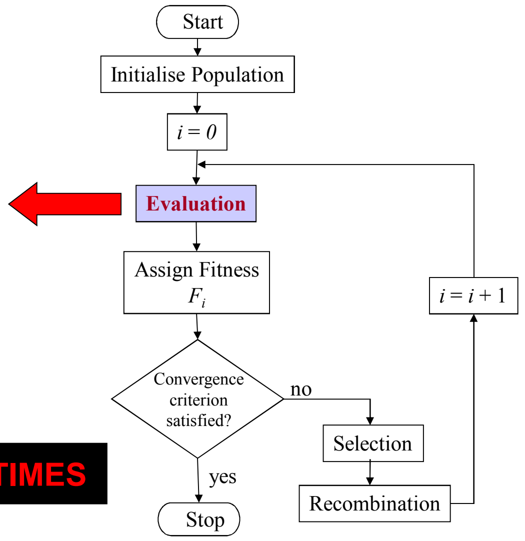
Computation time required to evaluate the solutions

Engineering problems:

Black Box
Numerical modelling routines

- Finite elements
- Finite differences
- Finite volumes
- etc

HIGH COMPUTATION TIMES



HYBRID ALGORITHMS

Three possible approaches to reduce the computation time

1. **During evaluation** – Some solutions can be evaluated using an approximate function, such as Fitness Inheritance, Artificial Neural Networks, etc (*this reduce the number of exact evaluations necessary*).

2. **During/after recombination** – Some individuals can be generated using more efficient methods (*this produce a fast approximation to the optimal Pareto frontier, thus the number of generations is reduced*).

3. **Local Search** – Some new individuals are generated by local search algorithms

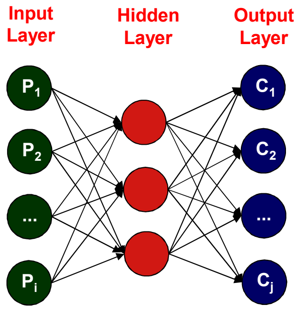
HYBRID ALGORITHMS

Three possible approaches to reduce the computation time

1. **During evaluation** – Some solutions can be evaluated using an approximate function, such as Fitness Inheritance, Artificial Neural Networks, etc (*this reduce the number of exact evaluations necessary*).

Using Artificial Neural Networks (ANN) to evaluate some solutions during the search.

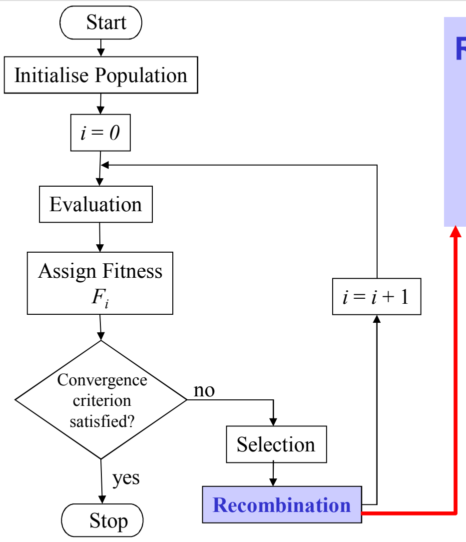
Artificial Neural Networks



Three possible approaches to reduce the computation time

2. **During/after recombination** – Some individuals can be generated using more efficient methods (*this produce a fast approximation to the optimal Pareto frontier, thus the number of generations is reduced*).

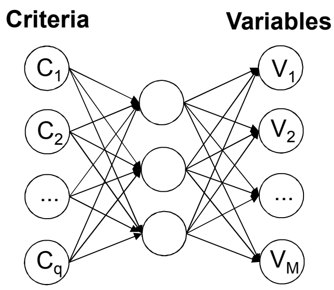
Use of an IANN together with the *Recombination* operators



Recombination operators:

- Crossover
- Mutation

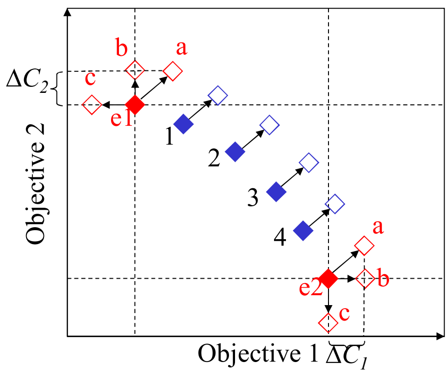
Inverse ANN (*IANN*)



Set of Solutions Generated with the IANN

Selection of *n+q* solutions from the present population to generate:

- 3.*q* extreme solutions
- *n* interior solutions



For $j = 1, \ldots, q$
(where, q is the number of objectives)

Points 1, 2, …, *n*: $C_j = C_j^{'} + \Delta C_j$

Point e_j to *a*: $C_j = C_j^{'} + \Delta C_j$

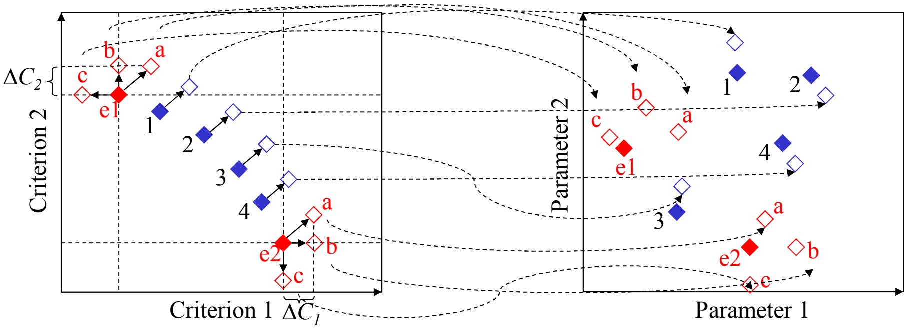
Point e_j to *b*: $C_{j(j=i)} = C_j^{'} \quad \wedge \quad C_{j(j \neq i)} = C_j^{'} + \Delta C_j$

Point e_j to *c*: $C_{j(j=i)} = C_j^{'} - \Delta C_j \quad \wedge \quad C_{j(j \neq i)} = C_j^{'} + \Delta C_j$

HYBRID ALGORITHMS

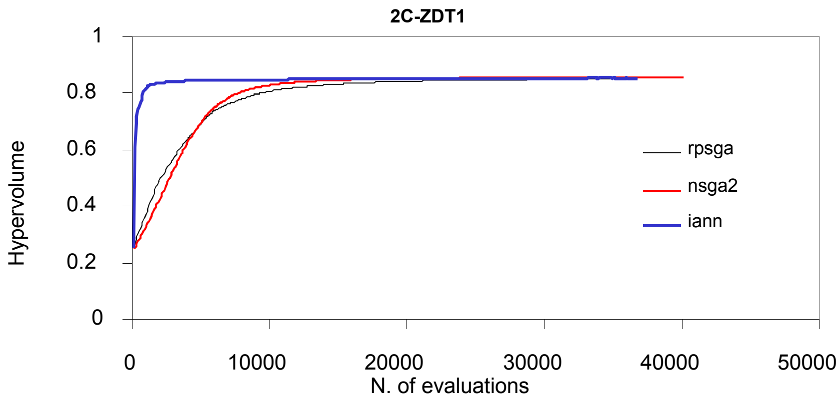
Set of Solutions Generated with the IANN

Use of IANN to generate new solutions



HYBRID ALGORITHMS

MOEA - Inverse ANN



- **The Inverse ANN approach has the largest improvement during the first generations, i.e., when the solution is far from the optimum;**

Three possible approaches to reduce the computation time

3. **Local Search** – Some new individuals are generated by local search algorithms

Filter Method

FILTER METHOD:

- Uses the concept of non-dominance to build a filter that accepts iterates that improve either the objective function or the constraints violation instead of a combination of the two measures.

$$\begin{array}{ll} \text{minimize} & F(x) \\ \text{subject to} & C(x) \geq 0 \end{array}$$

In the FM the main idea is to:

- Minimize a measure of the constraints violation:
- Minimize the objective function *F(x)*

$$\theta(x) = \left\| C(x)^{+} \right\| = \left\| max(C(x), 0) \right\|$$

The filter technique attempt to minimize both functions, but a certain emphasis is placed on the first measure, since a point has to be feasible in order to be an optimal solution.

HYBRID ALGORITHMS

Filter Method

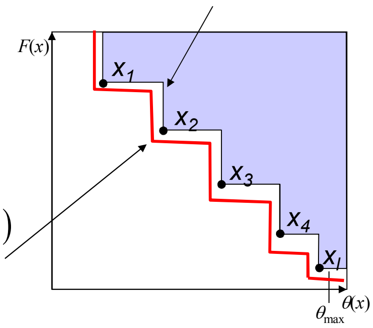
A filter is a finite set of pairs ($\theta(x_l)$; $F(x_l)$), that correspond to a collection of previous iterates x_l, with the additional requirement that a point in order to be accepted, has to improve at least one of the two measures when compared to those of the previous iterates.
In order words, a point x can be accepted only if:

$$\theta(x) \leq \theta(x_l) \; or \; F(x) \leq F(x_l)$$

To avoid the acceptance of pairs that are arbitrarily close to the boundary of the set of all pairs that are dominated by the filter, this condition is replaced by:
CONDITION 2

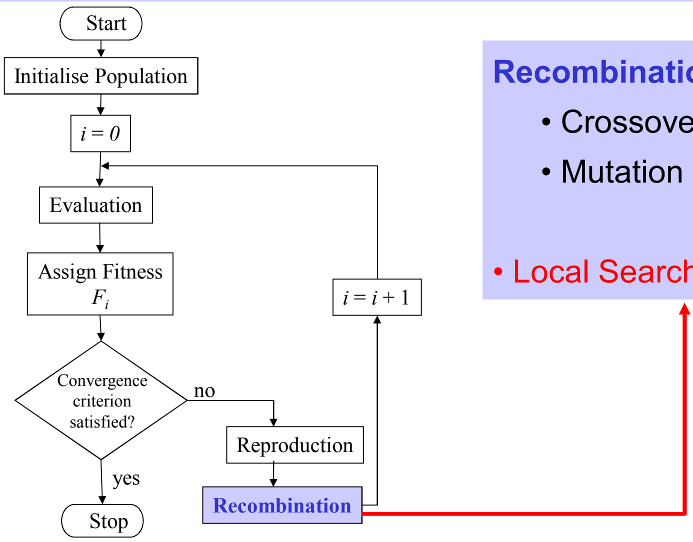
$$\theta(x) \leq (1-\gamma_\theta)\theta(x_l) \; or \; F(x) \leq F(x_l) - \gamma_F \theta(x_l)$$

$$where: \quad \gamma_\theta, \gamma_F \in [0,1]$$



HYBRID ALGORITHMS

Using a Local Search Procedure



Recombination operators:

- Crossover
- Mutation

- Local Search (PSFM)

HYBRID ALGORITHMS

Algorithm 1

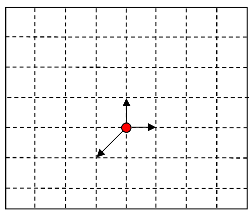
If generation = **Ngen** **then**
    Select the **Nsol** using the clustering technique
    Set S = { }
    for i=1, …, M (n. of objectives) **do**
        θ(x) = measure of restriction violation for all objectives (different of i)
        F(x) = f(i)
        Apply the PSFM (Algorithm 2) to get a new set of solutions - Si
        Set S = S ∪ Si
    end for
    Incorporate S, the solutions generated, in the main population
end if

HYBRID ALGORITHMS

Algorithm 2:

Define: α(tol)>0, α(0)>0, {d(1), d(2), …,d(2q)} and {x(1), x(2), …, x(**Nsol**)}

For i=1, …,Nsol **do**
    If i=1 **then** initialize the filter
    Set α = α(0) * Δα
    For j=0, …, q **do**
        Set x(i) = x(i) + α d(j)
    end for
    Evaluate θ and F
    If the pair (θ, F) do not belong to the Filter, **then**
        If CONDITION 2 holds **then**
            Accept the trial point (x(i))
            break
        end if
    else
        Reject the point
        Set α = ½ α.
        If α < α(tol) **then break**
    end if
end for



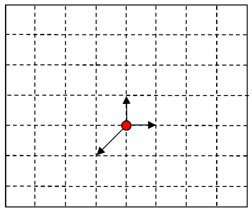
$$\Delta\alpha = max_{k=1,\dots,N}\left(u_k - l_k\right)/\varepsilon$$

- q is the number of variables
- u and l are the upper and lower bounds for each variable
- **ε** is a parameter to be defined

HYBRID ALGORITHMS

Algorithm 2:

Define: α(tol)>0, α(0)>0, {d(1), d(2), …,d(2q)} and {x(1), x(2), …, x(**Nsol**)}



For i=1, …,Nsol **do**
  If i=1 **then** initialize the filter
  Set α = α(0) * $\Delta\alpha$
  For j=0, …, q **do**
    Set x(i) = x(i) + α d(j)
  end for
  Evaluate θ and F
  If the pair (θ, F) do not belong to the Filter, **then**
    If CONDITION 2 holds **then**
      Accept the trial point (x(i))
      break
    end if
  else
    Reject the point
    Set α = ½ α.
    If α < α(tol) **then break**
  end if
end for

$$\Delta\alpha = max_{k=1,\dots,N}\left(u_k - l_k\right)/\varepsilon$$

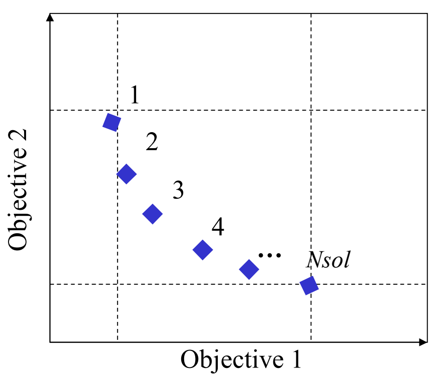
- q is the number of variables
- u and l are the upper and lower bounds for each variable
- ε is a parameter to be defined

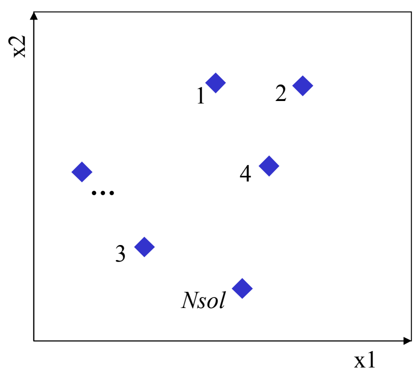
HYBRID ALGORITHMS

Set of Solutions Generated with PSFM

At a pre-defined generation (*Ngen*):

Selection of *Nsol* solutions from the present population





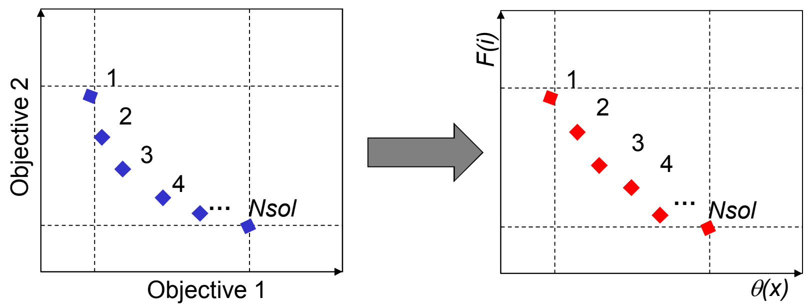
HYBRID ALGORITHMS

Set of Solutions Generated with PSFM

For each objective (for i=1, …, M):

θ(x) = measure of restriction violation for all objectives (different of i)

F(x) = f(i)



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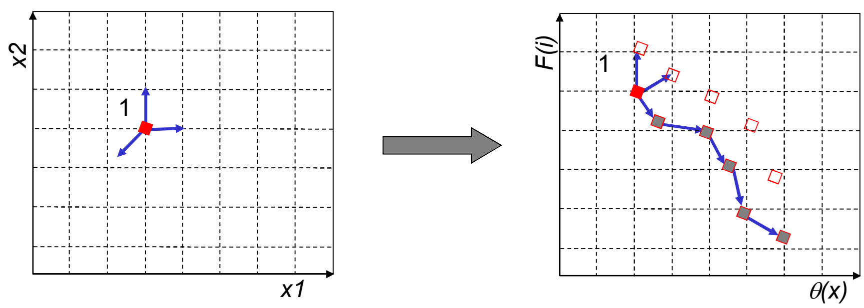
HYBRID ALGORITHMS

Apply the PSFM to generate a new set of solutions - S



$x_l = x_l + \alpha \, d_l$

$\alpha = max_{k=1,\dots,N}(u_k - l_k)/\varepsilon$

ε is a parameter to be defined

$\theta(x) \le (1-\gamma_\theta)\theta(x_l) \; or \; F(x) \le F(x_l) - \gamma_F \theta(x_l)$

$where: \quad \gamma_\theta, \gamma_F \in [0,1]$

Incorporate the solutions generated in the main population

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K. Deb et. al - Test Problem Generator

$$
\begin{aligned}
&\text{Minimize} \quad f_1(x_1),\\
&\text{Minimize} \quad f_2(x_2),\\
&\quad \vdots \quad \vdots\\
&\text{Minimize} \quad f_{q-1}(x_{q-1}),\\
&\text{Minimize} \quad f_q(x) = g(x_q)h(f_1(x_1), f_2(x_2), \ldots, f_{q-1}(x_{q-1}), g(x_q)),\\
&\text{Subject} \quad to \quad x_i \in \Re^{|x_i|}, \quad \text{for } i = 1, 2, \ldots, q.
\end{aligned}
$$

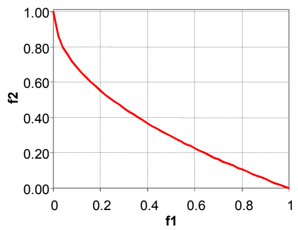
2 Criteria

2C-ZDT1 (*Convex*): $M = 30$; $x_i \in [0, 1]$

$$f_1(x_1) = x_1$$

$$f_2(x_2, \cdots, x_M) = g \times \left(1 - \sqrt{f_1 / g}\right)$$

$$\text{where, } g(x_2, \cdots, x_M) = 1 + 9\frac{\sum_{i=2}^{M} x_i}{M - 1}$$



HYBRID ALGORITHMS

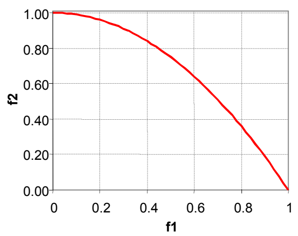
2 Criteria

2C-ZDT2 (*Non-convex*): $M = 30$; $x_i \in [0, 1]$

$$f_1(x_1) = x_1$$

$$f_2(x_2, \cdots, x_M) = g \times \left(1 - \left(f_1 / g\right)^2\right)$$

$$\text{where, } g(x_2, \cdots, x_M) = 1 + 9\frac{\sum_{i=2}^{M} x_i}{M - 1}$$

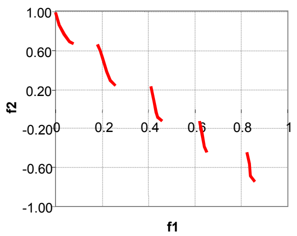


2C-ZDT3 (*Discrete*): $M = 30$; $x_i \in [0, 1]$

$$f_1(x_1) = x_1$$

$$f_2(x_2, \cdots, x_M) = g \times \left(1 - \sqrt{f_1 / g} - \left(f_1 / g\right)\sin(10\pi f_1)\right)$$

$$\text{where, } g(x_2, \cdots, x_M) = 1 + 9\frac{\sum_{i=2}^{M} x_i}{M - 1}$$



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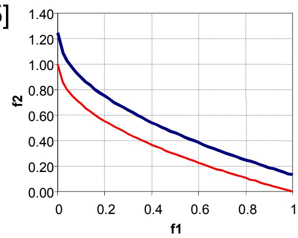
2 Criteria

2C-ZDT4 (*Multimodal*): $M = 10$; $x_1 \in [0, 1]$; $x_i \in [-5, 5]$

$$f_1(x_1) = x_1$$

$$f_2(x_2, \cdots, x_M) = g \times \left(1 - \sqrt{f_1 / g}\right)$$

$$\text{where, } g(x_2, \cdots, x_M) = 1 + 10(M-1) + \sum_{i=2}^{M} \left(x_i^2 - 10\cos(4\pi x_i)\right)$$

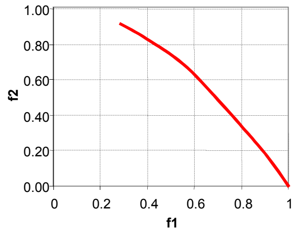


2C-ZDT6 (*Non-uniform*): $M = 10$; $x_i \in [0, 1]$

$$f_1(x_1) = 1 - \exp(-4x_1)\sin^6(6\pi x_1)$$

$$f_2(x_2, \cdots, x_M) = g \times \left(1 - \left(f_1 / g\right)^2\right)$$

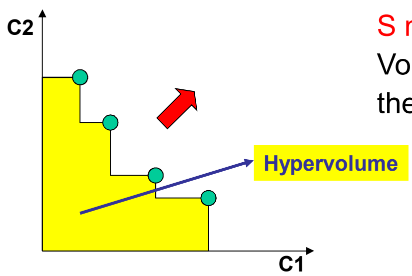
$$\text{where, } g(x_2, \cdots, x_M) = 1 + 9\left(\frac{\sum_{i=2}^{M} x_i}{M-1}\right)^{0.25}$$



HYBRID ALGORITHMS

Hypervolume Metric

This metric calculates the dominated space volume, enclosed by the nondominated points and the origin.



Criteria C1 and C2 to maximize

S metric:
Volume of the space dominated by the set of objective vectors

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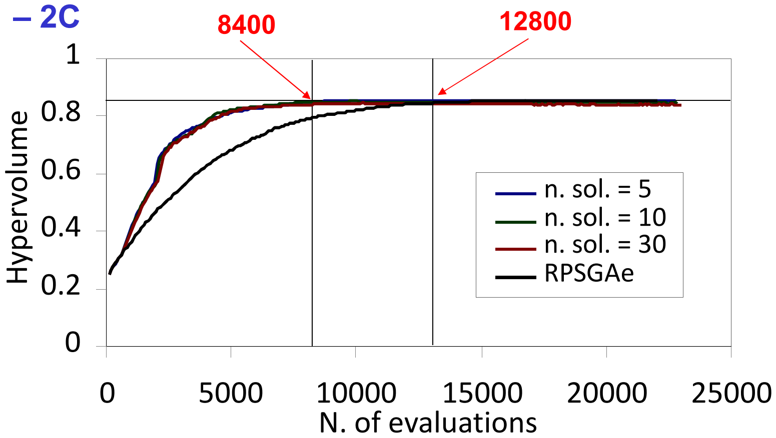
HYBRID ALGORITHMS

Comparison between RPSGA and MO-PSFM: Influence of the n. of solutions

ZDT1 – 2C

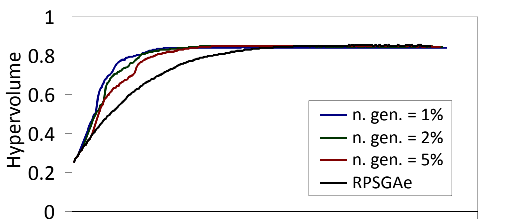


Average of 30 runs with different seed values

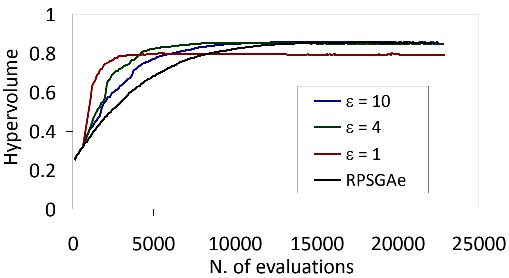
HYBRID ALGORITHMS

Comparison between RPSGA and MO-PSFM

ZDT1



Influence of the n. of generations

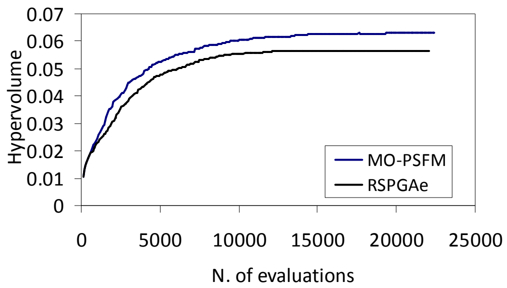


Influence of ε

HYBRID ALGORITHMS

Comparison between RPSGA and MO-PSFM

ZDT4 – 2C



HYBRID ALGORITHMS

Comparison between RPSGA and MO-PSFM

Test problem	Hypervolume	RPSGAe	MO-PSFM	Improvement (%)
ZDT1-2C	0.854	12800	8400	**34**
ZDT2-2C	0.773	11900	8600	**28**
ZDT3-2C	2.06	21300	13200	**38**
ZDT4-2C	**Not applicable**			
ZDT6-2C	0.66	15500	4030	**74**

HYBRID ALGORITHMS

- A Memetic multi-objective algorithm, based in the incorporation of a PSFM within a MOEA, was proposed (MO-PSFM).
- The results produced, using some difficult test problems, indicate that the hybrid methodology proposed is able to reduce the number of evaluations of the objective functions necessary to get identical performance.
- Since the local search methodology used here is very simple there is some room for improvements in the MO-PSFM.

HYBRID ALGORITHMS

Comparison between the different methods

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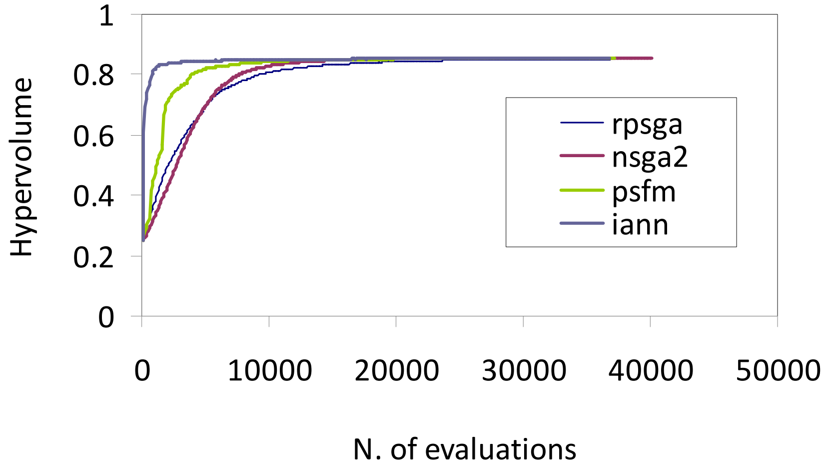
HYBRID ALGORITHMS

ZDT1: rpsga, nsga-II, iann, psfm



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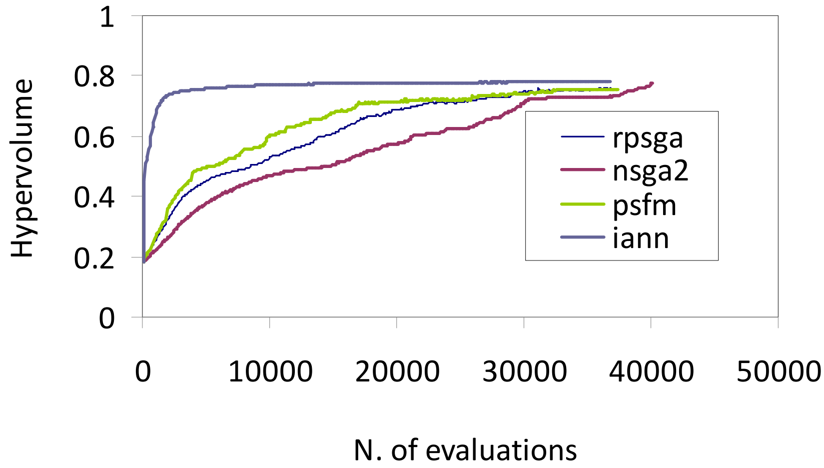
HYBRID ALGORITHMS

ZDT2: rpsga, nsga-II, iann, psfm



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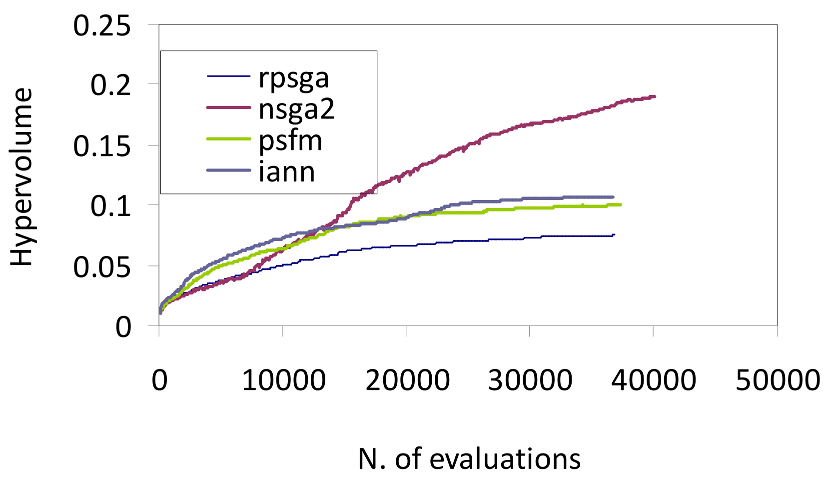
HYBRID ALGORITHMS

ZDT4: rpsga, nsga-II, iann, psfm



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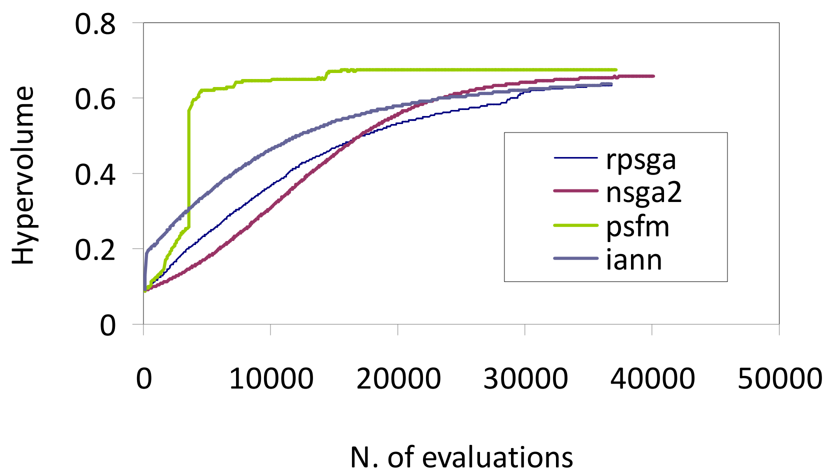
HYBRID ALGORITHMS

ZDT6: rpsga, nsga-II, iann, psfm



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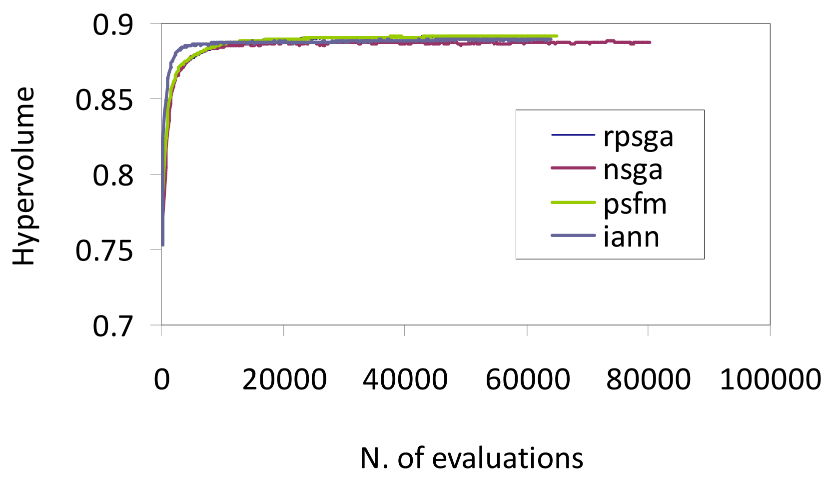
HYBRID ALGORITHMS

DTLZ2: rpsga, nsga-II, iann, psfm



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HYBRID ALGORITHMS - References

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4- F. Mendes, V. Sousa, M.F.P. Costa, A. Gaspar-Cunha, Multi-Objective Memetic Algorithm using Pattern Search Filter Methods, EU/Meeting 2009, Porto, Portugal, April 2009.

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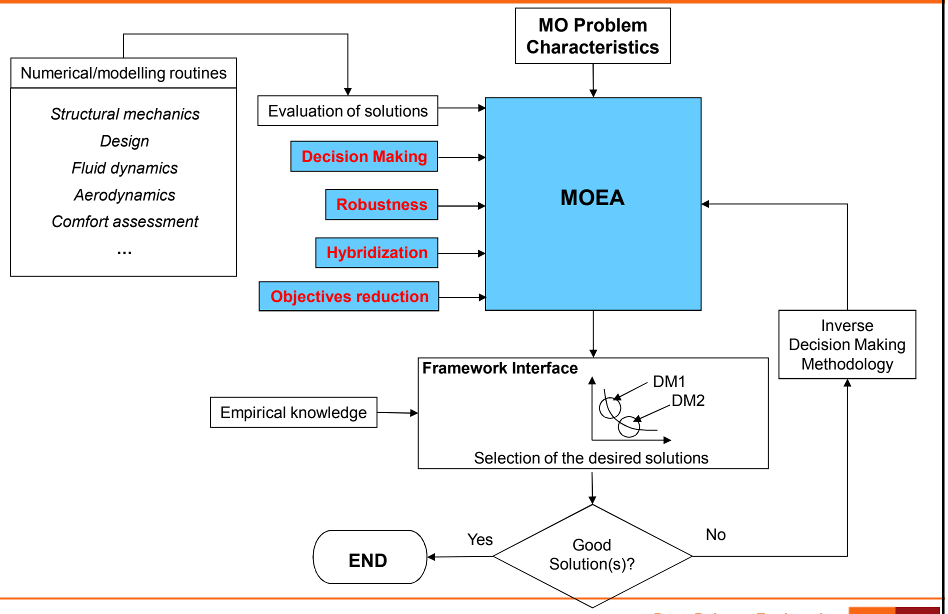
HYBRID ALGORITHMS



NUMBER OF OBJECTIVES REDUCTION

- Another important issue consists in verifying if all the objectives considered for the resolution of the problem are important for obtaining a good solution.
- A technique for reducing the number of objectives is, for that reason, of high importance.
- The reduction of the number of objectives will facilitate both the optimization and the decision phases.

NUMBER OF OBJECTIVES REDUCTION

- D. Brockhoff and E. Zitzler, Dimensionality Reduction in Multiobjective Optimization: The Minimum Objective Subset Problem, Operations Research Proceedings, 2006.
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- D. Brockhoff and E. Zitzler, Objective Reduction in Evolutionary Multiobjective Optimization, Evolutionary Computation}, 17, 2, 135-166, 2009.

NUMBER OF OBJECTIVES REDUCTION

- K. Deb, D. Saxena, Searching for pareto-optimal solutions through dimensionality reduction for certain large-dimensional multi-objective optimization problems. Proceedings of the IEEE Congress on Evolutionary Computation (CEC2006), 3353-3360. IEEE Computer Society Press, Los Alamitos, 2006.
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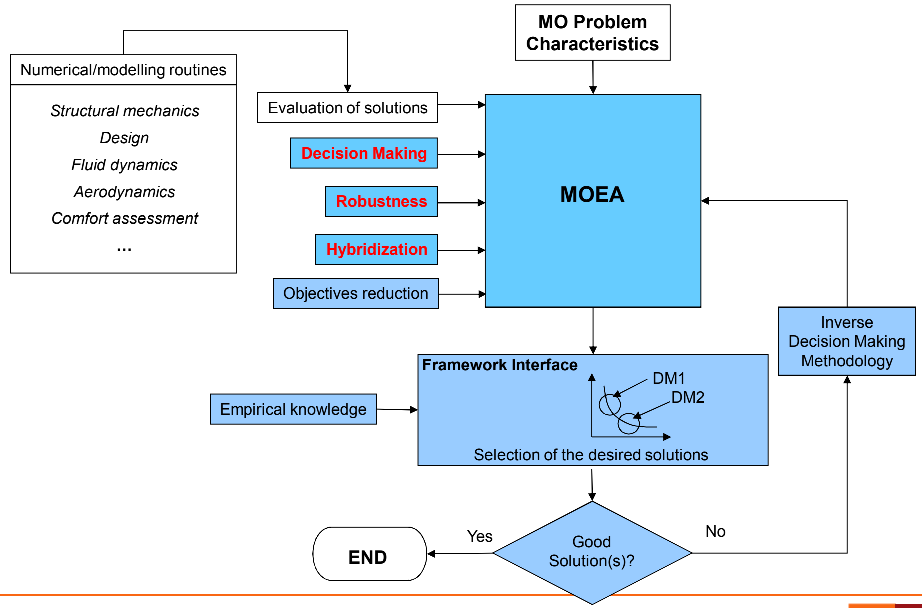
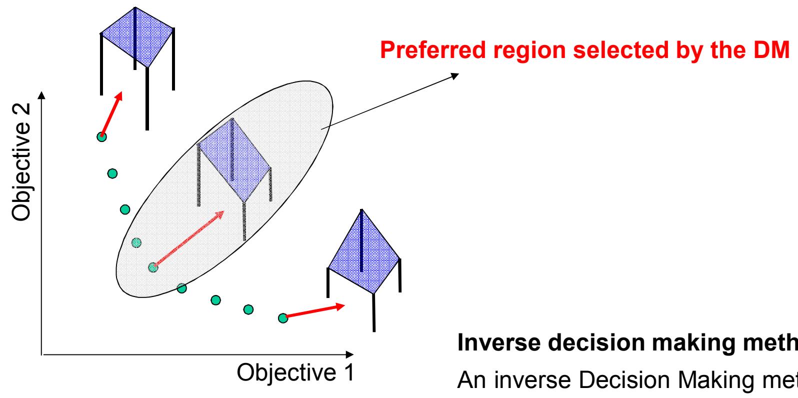
CASE STUDY



CASES STUDY

Design of a roof structure

CASE STUDY



Inverse decision making methodology

An inverse Decision Making methodology was applied to determine the weights corresponding to the region of the search space selected by the DMs. This information was incorporated on the MOEA to generate new solutions.

Framework interface

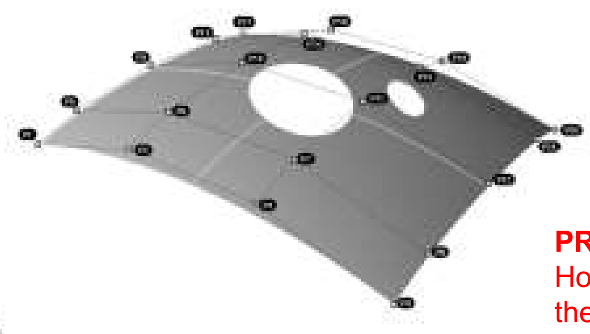
At this phase the solutions obtained are shown to the various DMs involved on the process.

CASE STUDY

Design of a roof structure *(illustrative problem)*

The aim is to define the best location of the points by:

1. Minimizing the cost (quantified by the area)
2. Minimizing the Day Light



PROBLEM:
How to define the limits of the design parameters?

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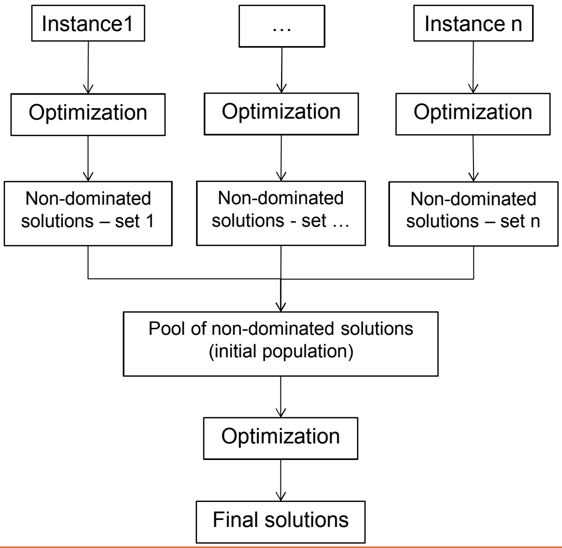
CASE STUDY

Strategy adopted



CASE STUDY

Instance 1: the coordinates of the 20 control points (corresponding to 60 decision variables) the 3D coordinates are allowed to vary between 0.5 and 5 meters

Instance 2: the corners of the structure are fixed, i.e., points P1(0,0,0), P4(5,0,0), P17(0,5,0) and P20(5,5,0). In this case 48 decision variables are to be optimized.

Instance 3: the corners points as well the border points are fixed, i.e., points P1(0,0,0), P2(1.6,0,0.5), P3(0.338,0,0.5), P4(5,0,0), P8(5,0.65,0.18), P13(0,0.335,0.18), P16(5,0.335,0.18), P17(0,5,0), P18(1.6,5.0,0.5), P19(0.338,5,0.5) and P20(5,5,0). This corresponds to 24 decision variables.

In instances 2 and 3 the coordinates of the remaining control points are allowed to range in the interval [0.5, 5] meters (as in instance 1).

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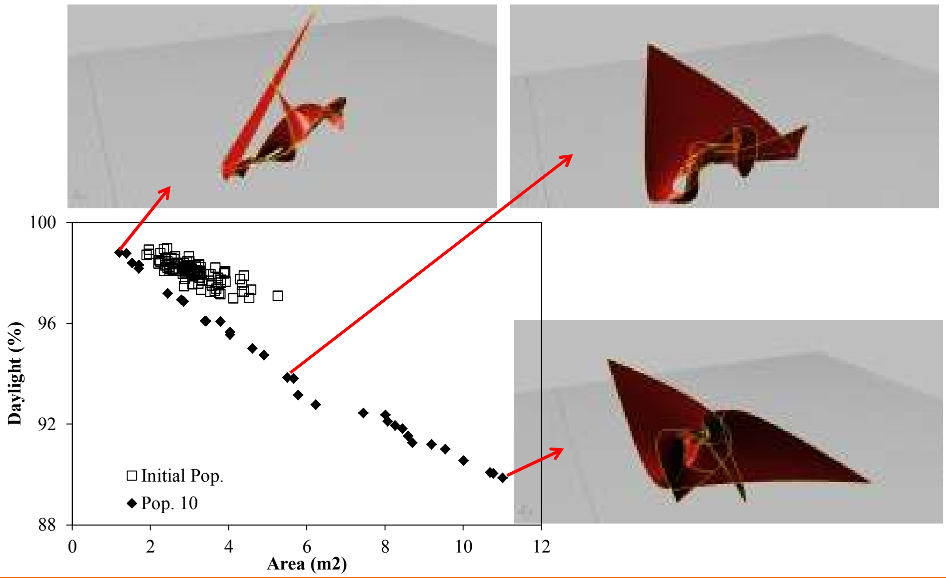
CASE STUDY

Instance 1



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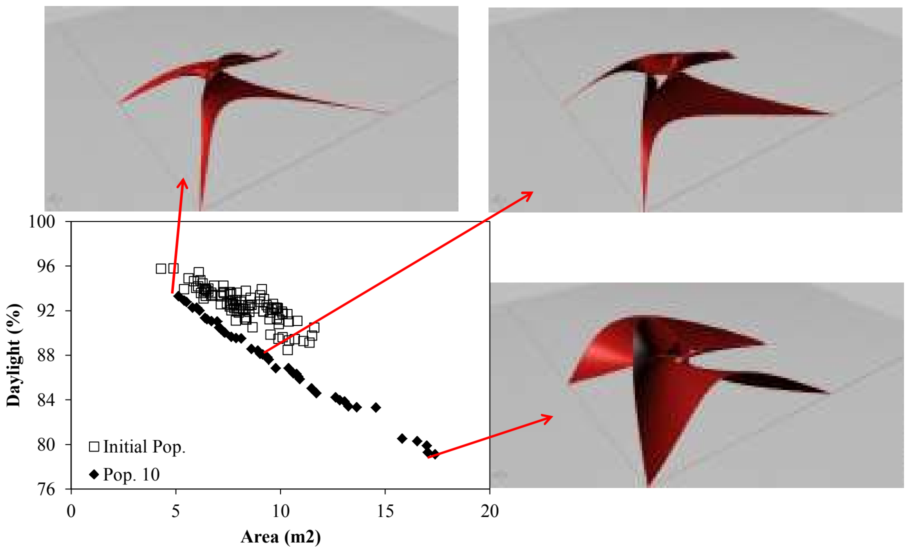
CASE STUDY

Instance 2



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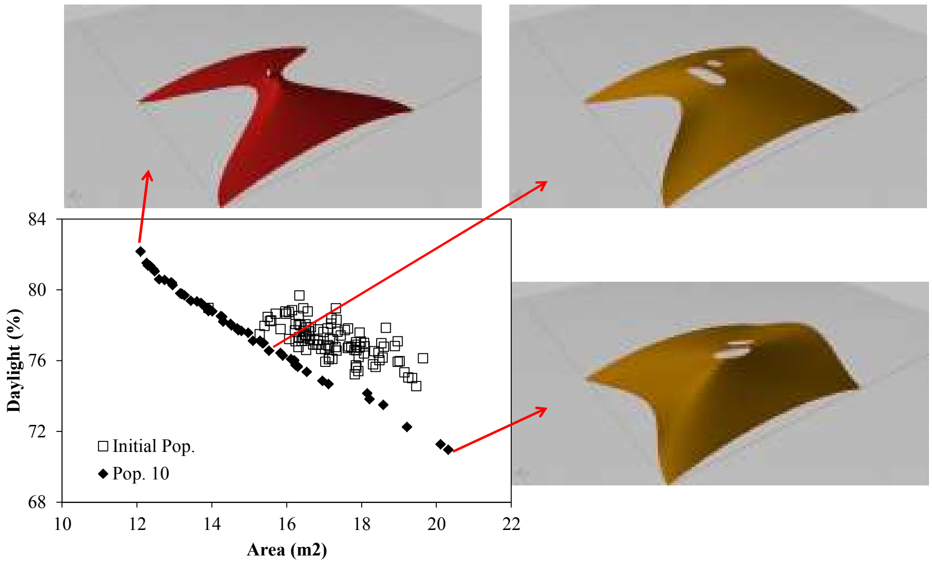
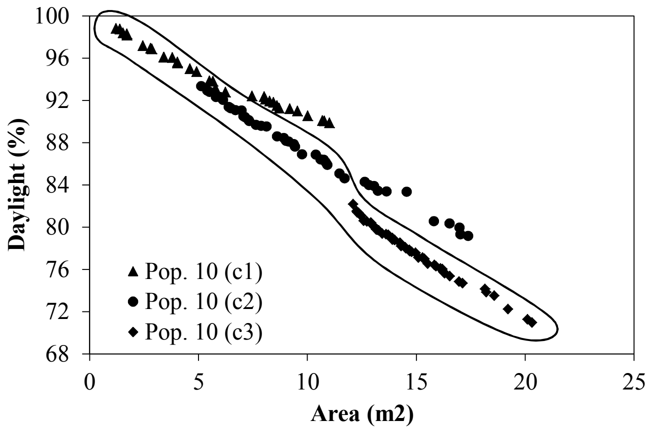
CASE STUDY

Instance 3



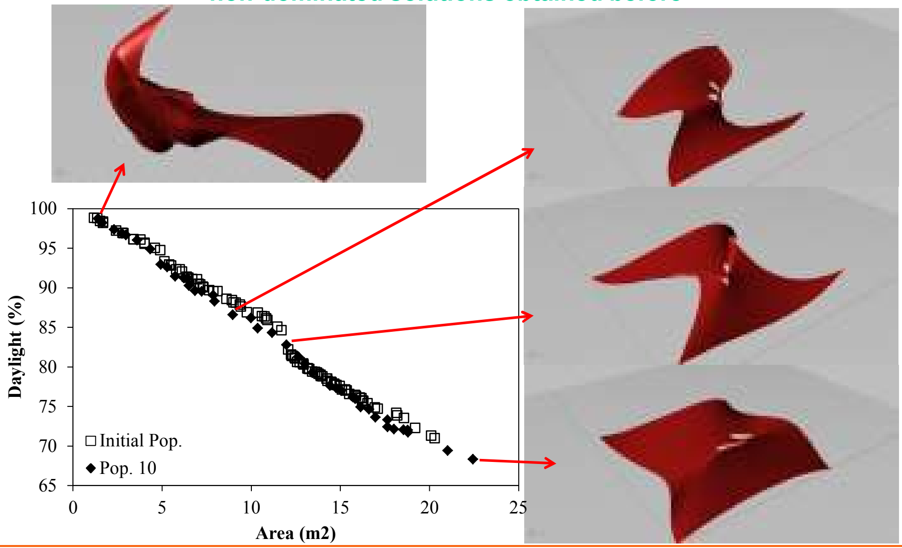
CASE STUDY

Optimal Pareto frontiers for Instances 1 to 3



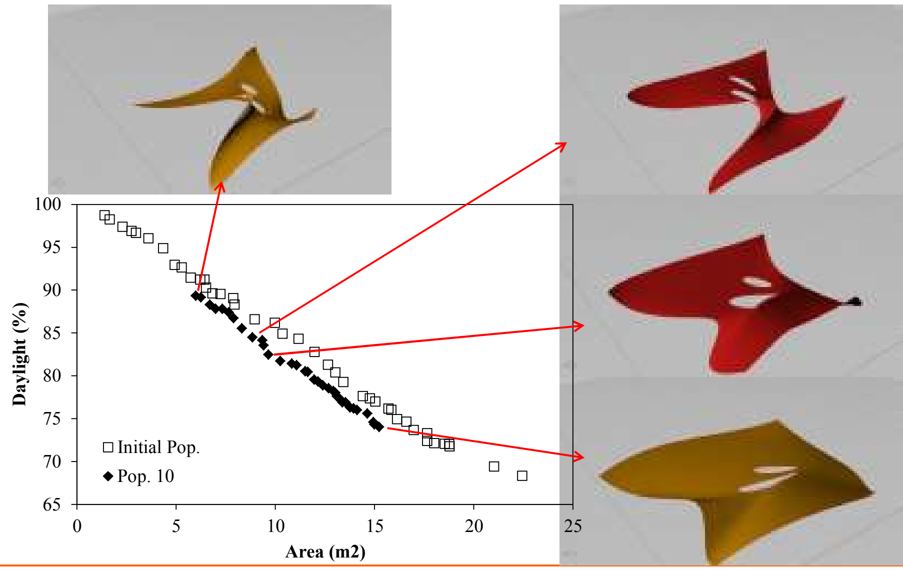
CASE STUDY

Optimal Pareto frontier using as initial population the non-dominated solutions obtained before



CASE STUDY

Optimal Pareto frontier using the DM strategy



GLOBAL CONCLUSIONS

- A Multi-Objective Multidisciplinary Design Optimization (MO-MDO) approach was proposed to deal with the practical complexities of large multi-objective problems.
- The methodology links a MOEA to decision making and robustness strategies that are able to assist the decision maker in selecting the best solutions that satisfy his preferences and/or are sufficiently robust against changes of the values of the decision variables, respectively.
- Also, with the aim of reducing the computation time often required by the evaluation routines (by decreasing the number of required real function evaluations) two different Memetic algorithms were suggested.
- Application of the methods to a few test problems demonstrated their effectiveness.